The Meaning of a Day

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“Day” means *something*. What is it?

Whether on Earth, Mars or the moons of Jupiter, the word should mean the same thing.

ITU Proposal to “cease leap seconds”

- Phrasing the question this way misses the whole point
- Leap seconds are a means to an end
- Atomic Time is a timescale constructed upward from the SI-second
- Civil Time is a timescale that flows downward from subdividing the Solar day
- The ITU proposal does not improve access to Atomic Time, it removes access to Solar time

Q: What is happening here?

(movie of the Moon's complex monthly phases and motions)

NASA Goddard Science Visualization Studio

http://svs.gsfc.nasa.gov/vis/a000000/a003800/a003894/

A: The Moon is actually just quietly spinning like a top

Like a top there are interesting superimposed motions such as precession that need to be considered over the long term, but all the effects seen in the movie are purely the result of our point of view as observers.

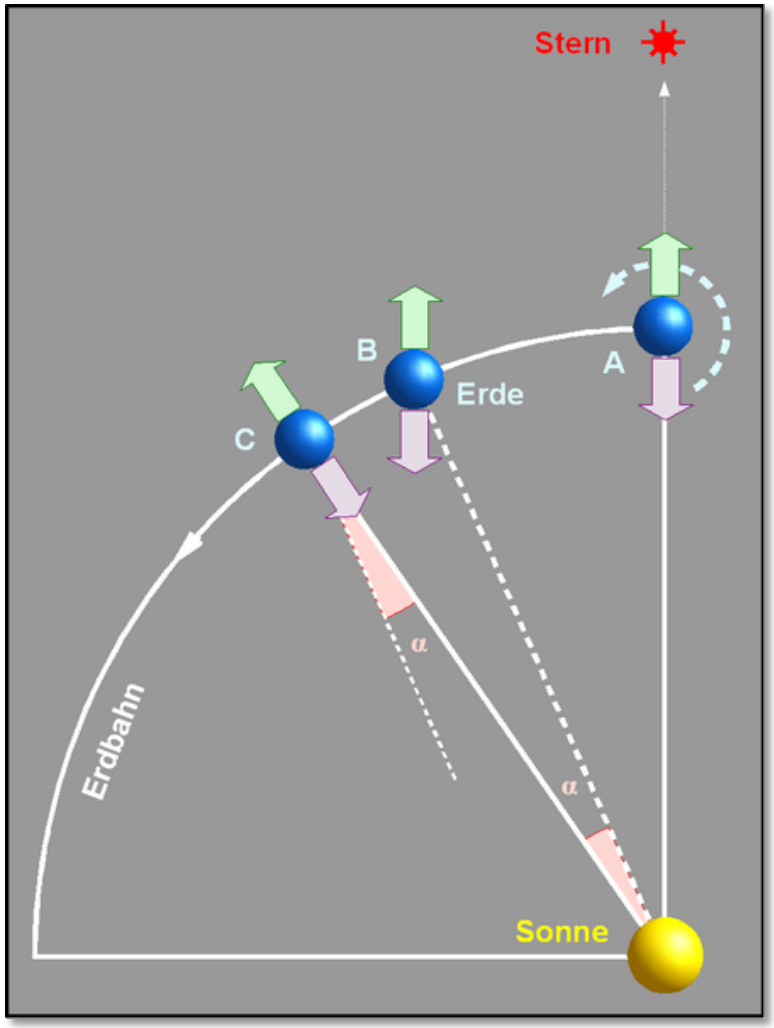
A day on the Moon

- "Day" on the Moon means exactly the same thing as day on Earth
- That is, one rotation relative to the Sun that illuminates its surface
- There is a simple relationship between the steady rotation of each relative to the stars and the corresponding length of day
- That the sidereal periods are slowly evolving *(with one result being leap seconds in UTC)* is simply a fact of life in the Solar System

Definition: *Synodic*

- For our purposes, the synodic day is the time to rotate relative to the Sun (A-C)

- As opposed to the sidereal day, relative to the stars (A-B)



“Sidereal Day” is an oxymoron

- Referring to sidereal days is largely meaningless
- And only works at all on some worlds
- Rather, this is the “sidereal rotation period”
- Versus the “solar day”

The take-away from this talk

1. "Day" means "Mean Synodic Day"

   (not whatever we or the ITU want it to mean)

2. The number of days in a year is one less than the number of rotations

   (due to lapping the Sun)

- Caveats: planets and moons that are
  - Not too far from spherical
  - Solid body rotators
  - Single sun

In the Solar System

There are 25^{+} such worlds:

- Four terrestrial planets
- Two dwarf planets (interior to Pluto)

  (sunlight on Pluto is ~250 full moons)
- Nineteen Moons (rounded by hydrostatic equilibrium)
  - our Moon
  - 17 around the gas giants
  - Charon

Planets and dwarf planets

	Object	Year *(Earth-days)*	Rotational Period *(d)*	Rotations per Year	Days per Year	Day *(Earth-days)*	Delta *synodic – sidereal*
	Mercury	87.97	58.6467	*1.5000*	*0.5000*	175.94	117 days
retro	Venus	224.70	-243.02	-0.9246	-1.9246	-116.75	-127 days
	Earth	365.25	*23:56:04*	366.25	365.25	*24:00:00*	*3:56*
	Moon	"	27.32	13.37	12.37	29.53	2.21 days
	Mars	687	*24:37:23*	669.6	668.6	*24:39:35*	*2:12*
	Ceres	1680	0.3781	4442.7	4441.7	0.3782	9 sec
retro	Pluto	90465	-6.387	-14164	-14165	-6.387	-39 sec

The number of days in a year is one less than the number of rotations

(Works for retrograde rotators, too! Just set rotational period negative.)

Day means “synodic day”

- On slow rotating worlds with large deltas between the sidereal period and the solar day
- On distant worlds with small deltas
- On worlds that rotate retrograde
- On worlds that rotate in orbital resonances
- On worlds like Mars where humans are already carrying out daily research projects
- It is the only coherent meaning for “Day”

Really just the Beat Frequency

Year/Day = Year/Rot – 1

1/Day = 1/Rot – 1/Year

$$f_{\text{diurnal}} = f_{\text{rotational}} - f_{\text{annual}}$$

Jupiter's Galilean Moons

Object	Year *(Earth-days)*	Rotational Period *(d)*	Rotations per Year	Days per Year	Day *(Earth-days)*	Delta *(seconds)*
Io	4332.59	1.76914	2448.98	2447.98	1.76986	62
Europa	"	3.55118	1220.04	1219.04	3.55409	251
Ganymede	"	7.15455	605.57	604.57	7.16638	1022
Callisto	"	16.68902	259.61	258.61	16.75355	5575

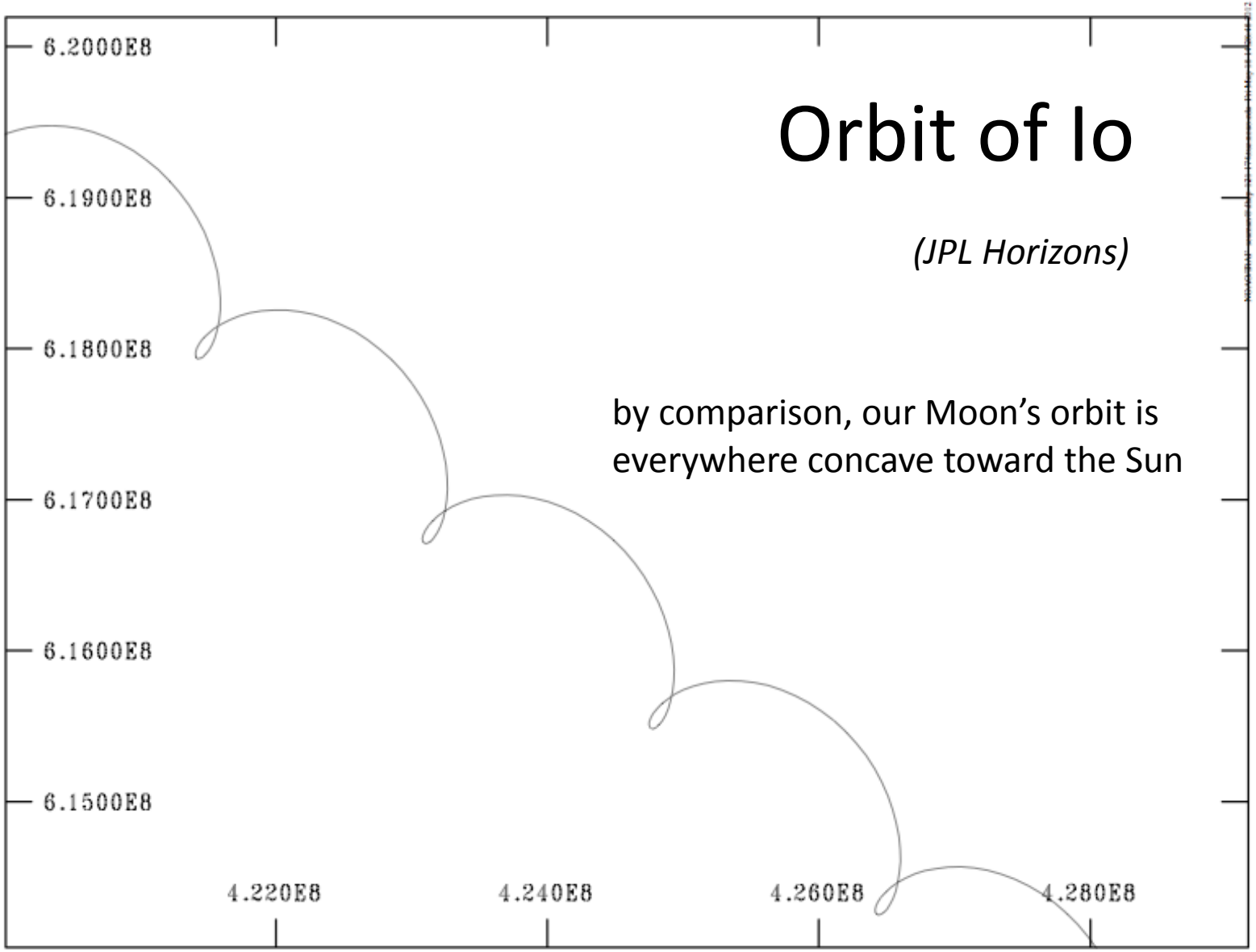
- All these moons are tidally locked
- Gas giants subtend much of the sky, but that doesn't affect the meaning of day
- Io, Europa & 4 inner moons of Saturn orbit faster than planet does about the Sun
- Delta *increases* for moons that are farther away

If the Moon were Jupiter

If the Moon were Saturn

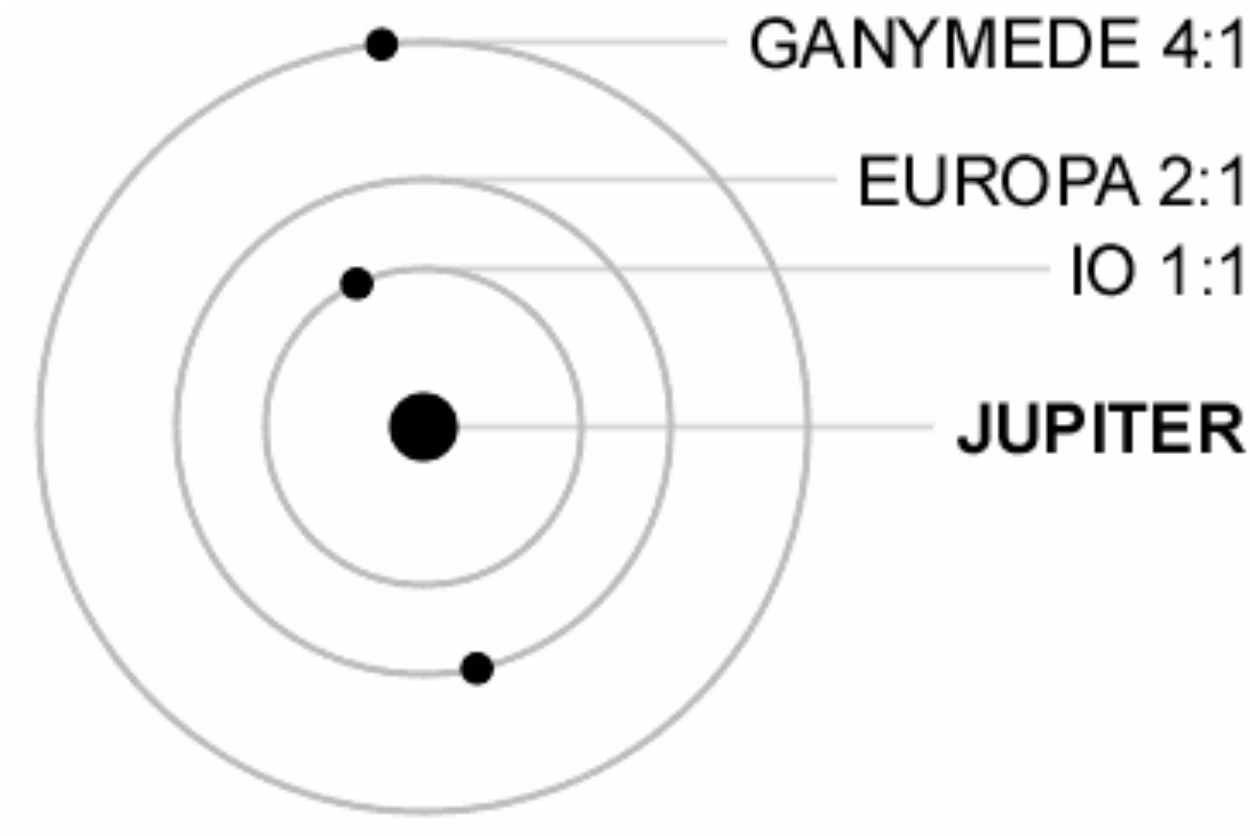
Orbit of Io

(JPL Horizons)



by comparison, our Moon's orbit is everywhere concave toward the Sun

Laplace Resonance



Length of day is coupled between the different worlds!

Days on Jupiter's moons

- The moons' rotations are tidally locked to Jupiter, and
- Their mean longitudes are locked to each other

$$\Phi_L = \lambda_{\mathrm{Io}} - 3 \cdot \lambda_{\mathrm{Eu}} + 2 \cdot \lambda_{\mathrm{Ga}} = 180^\circ$$

(applies to both sidereal and solar periods)

- So the days are locked and…

  A clock on one tells time on the others

Day means “synodic day”

- On tidally locked moons
- On distant moons with large deltas
- On moons orbiting so close their planet fills the sky
- On moons that move backwards in their shared planetary orbits
- On moons experiencing complex resonances
- …and it is the synodic day that permits clocks on one moon to be used to tell time on another

Saturn's Major Moons

Object	Year *(Earth-days)*	Rotational Period *(d)*	Rotations per Year	Days per Year	Day *(Earth-days)*	Delta *(seconds)*
Mimas	10759.22	0.94242	11416.59	11415.59	0.94250	7
Enceladus	"	1.37022	7852.18	7851.18	1.37039	15
Tethys	"	1.88780	5699.34	5698.34	1.88813	29
Dione	"	2.73692	3931.14	3930.14	2.73762	60
Rhea	"	4.51821	2381.30	2380.30	4.52011	164
Titan	"	15.945	674.77	673.77	15.969	2071
Iapetus	"	79.322	135.64	134.64	79.911	50890

Calendar days are synodic days

- All of this interesting behavior derives from the synodic day
- In particular, on Earth = mean solar time
- Calendars count the days
- Clocks divide them up

Moons of Uranus, Neptune, Pluto

	Object	Year *(Earth-days)*	Rotational Period *(d)*	Rotations per Year	Days per Year	Day *(Earth-days)*	Delta *(seconds)*
on its side	Miranda	30685.4	1.414	21701	21700	1.414	6
	Ariel	"	2.52	12177	12176	2.52	18
	Umbriel	"	4.144	7405	7404	4.145	48
	Titania	"	8.706	3525	3524	8.708	213
	Oberon	"	13.46	2280	2279	13.47	510
retro	Triton	60189	-5.877	-10241	-10242	-5.876	-50
locked	Charon	90465	-6.387	-14164	-14165	6.387	-39

A clock on Pluto can be used to tell time on Charon

Aside: How long is a day on the Sun?

$$f_{\text{day}} = f_{\text{rot}} - f_{\text{year}}$$

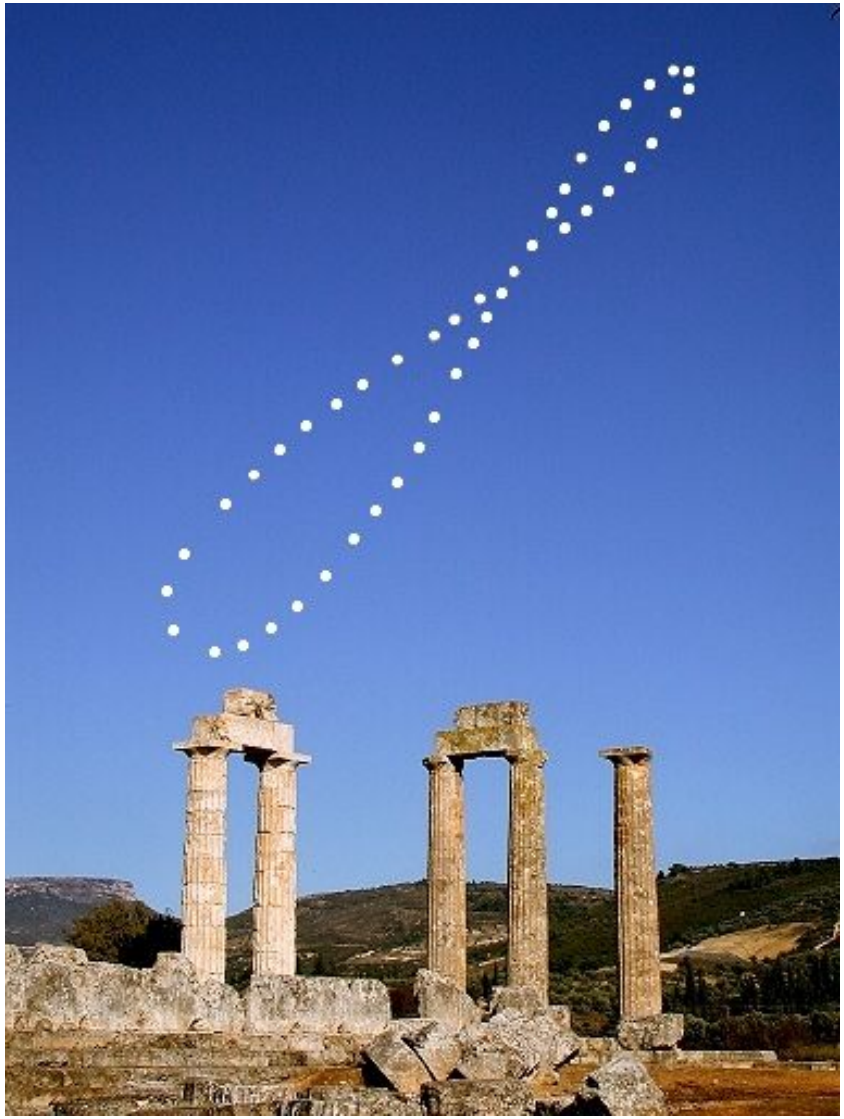
- The year is infinite, so the day = one rotation
  - But the Sun is self-illuminated
  - And rotates differentially
- *Debate*:
  - Infinite?
  - Zero?
  - Undefined?

What about apparent solar time?

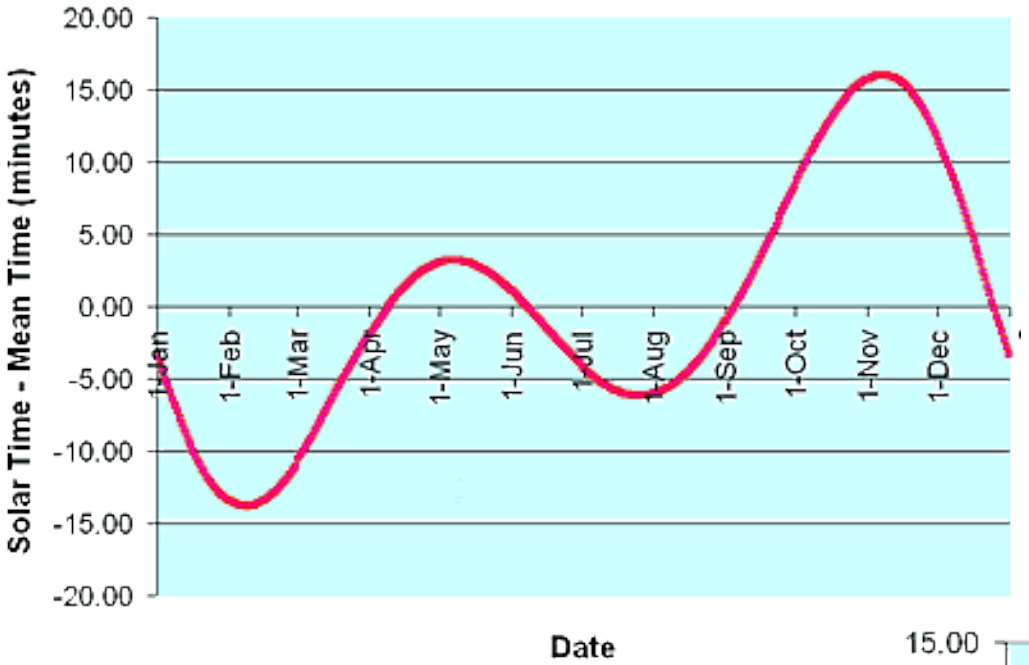
- Apparent solar time is not the "real day"
  - Rather, apparent time is just an accumulated daily deviation
- Mean solar time is not an average
  - It may be calculated that way, but it isn't what mean days "mean"

Consider the Analemma (*gr*: “sundial”)

– Sun’s position at the same time-of-day over a year

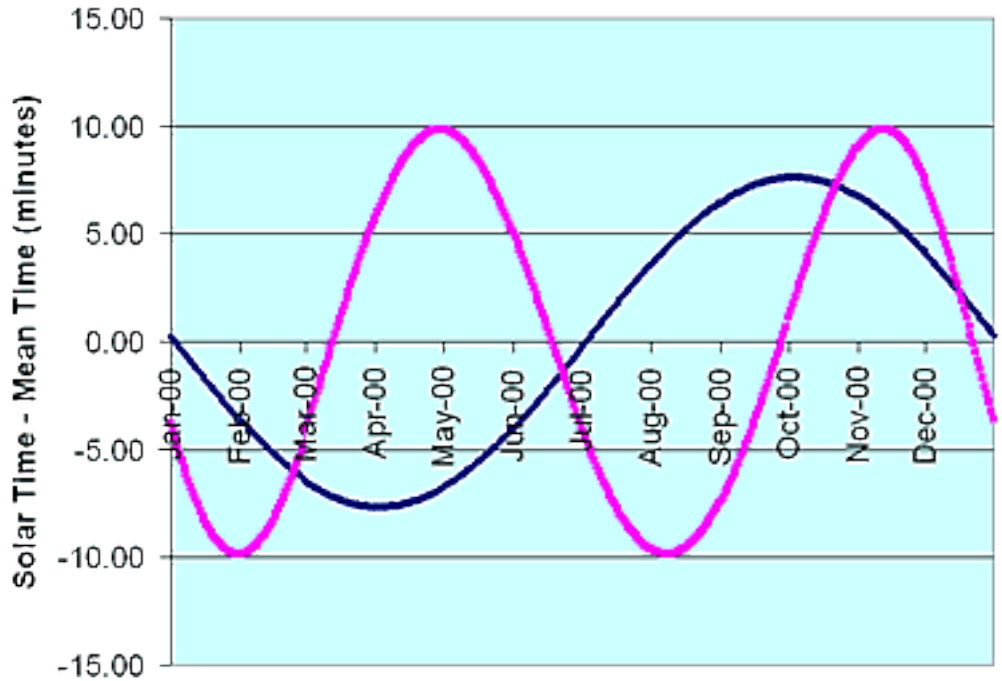


Zeus' Temple, ancient Nemea, 14h UT – A. Ayiomamitis

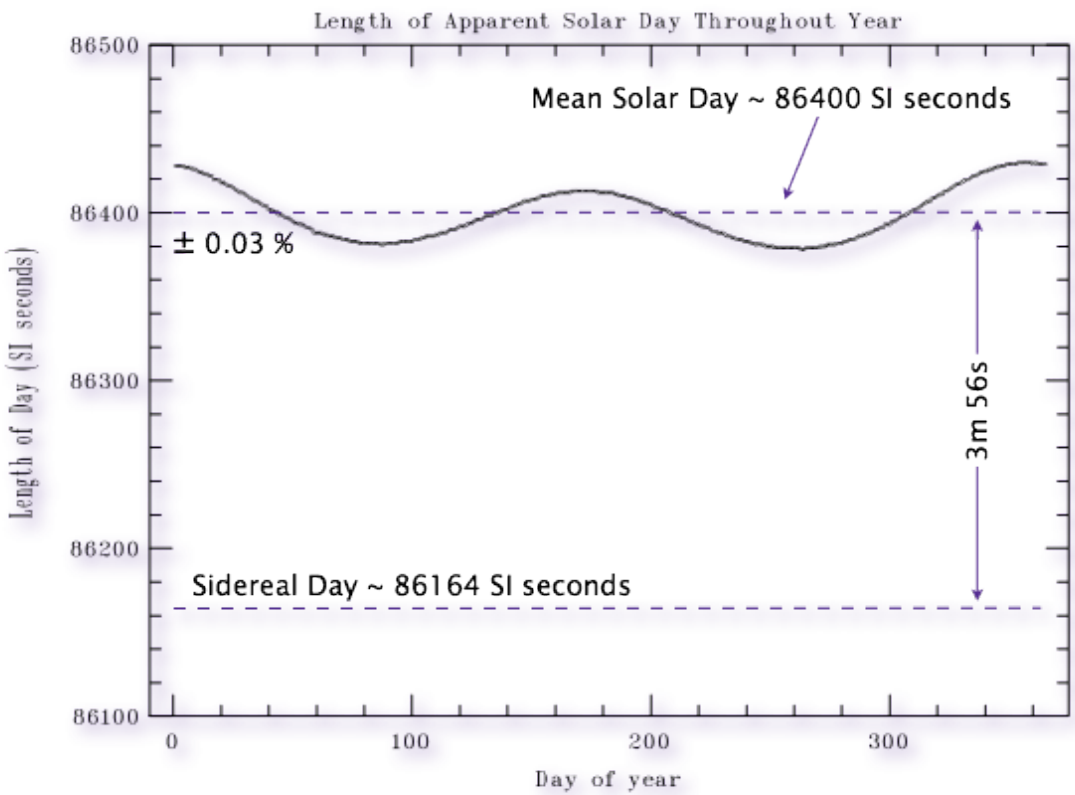


An analemma is a two dimensional depiction of the Equation of Time

Equation of Time is due to obliquity of ecliptic plus orbital eccentricity



So how long is a Day?



- Sidereal days are constant length
  (albeit slowly varying)
- Each solar day differs slightly
- Daily residuals accumulate as **Equation of Time**
- Daily ms level epsilon from 1820 epoch
- ε's accumulate as **Leap Seconds**

Meanwhile on Neptune

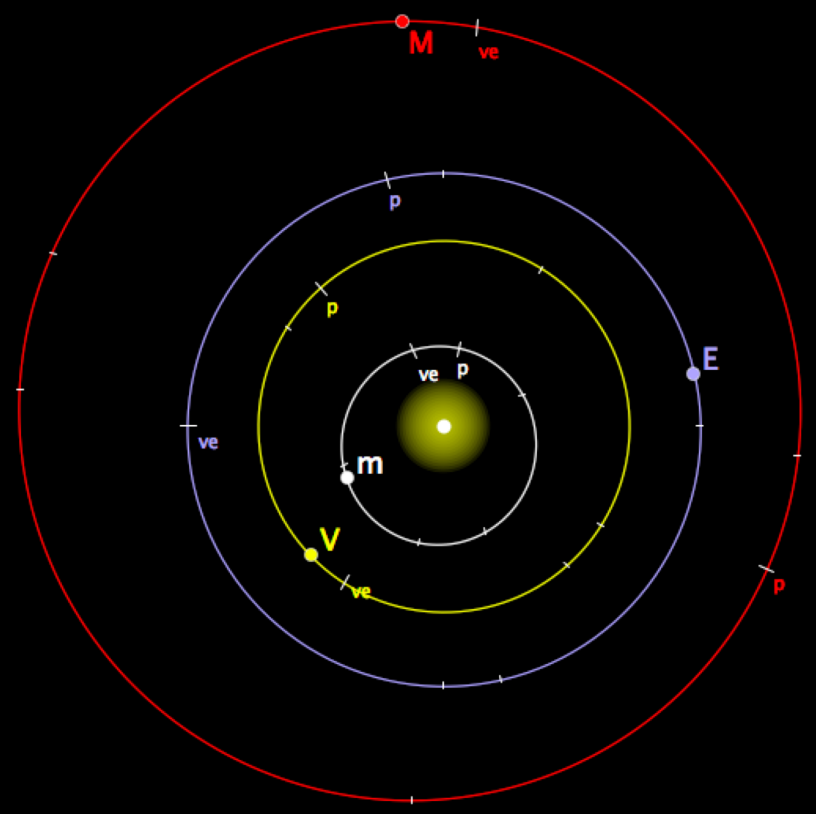
(movie tracing out Neptune's classic figure eight analemma)

http://www.analemma.com/Pages/OtherAnalemmas/OtherPlanetAnalemmas/NeptuneMovie.html

Mars' orbit is more eccentric

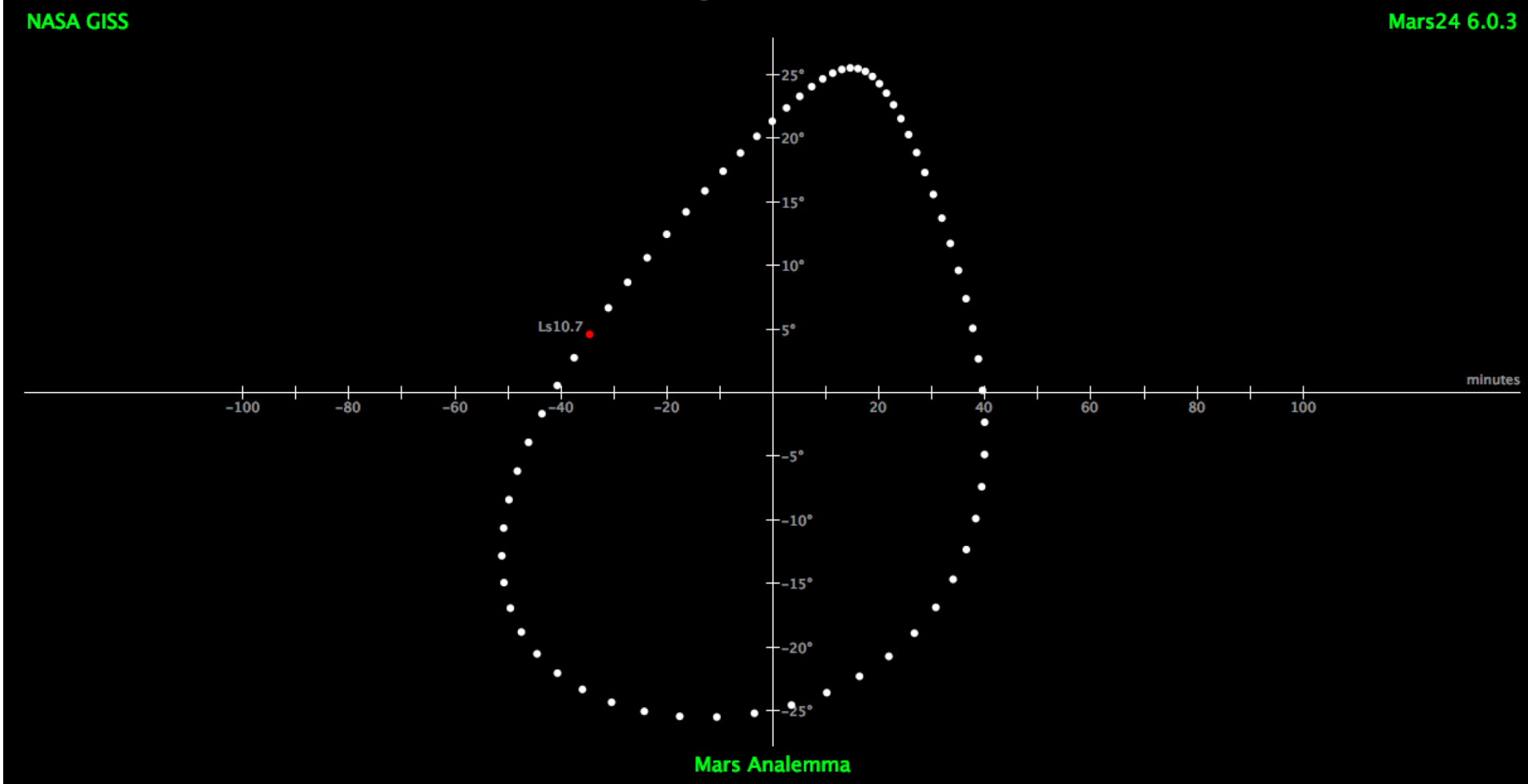
NASA GISS

Mars24 6.0.3



Orbital Positions of the Inner Planets

So the equation of time makes larger excursions



What kind of thing is civil time?

- It is an angle

  and

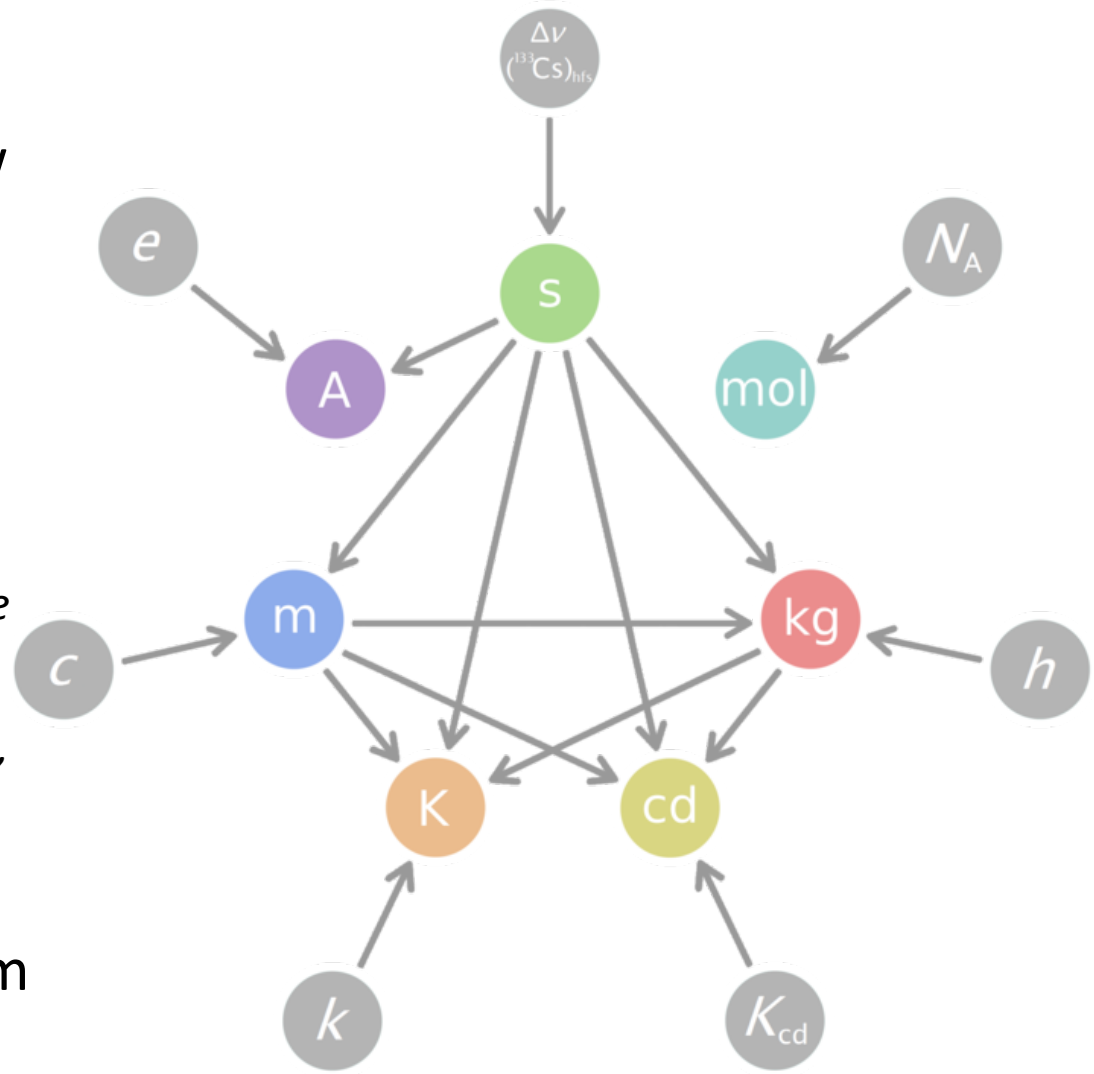
- It is duration

- Else what kind of quantity would (UT1 – UTC) be?

- Solar time is measured with a protractor
- Atomic time is a metronome

Metric System (SI) is also being redefined

- The 7 SI base units are being re-layered on a new set of fundamental constants
- The SI-second is:
  - *"9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom"*
- The frequency of Cs-133 is the keystone of the system



http://dx.doi.org/10.6028/jres.116.022

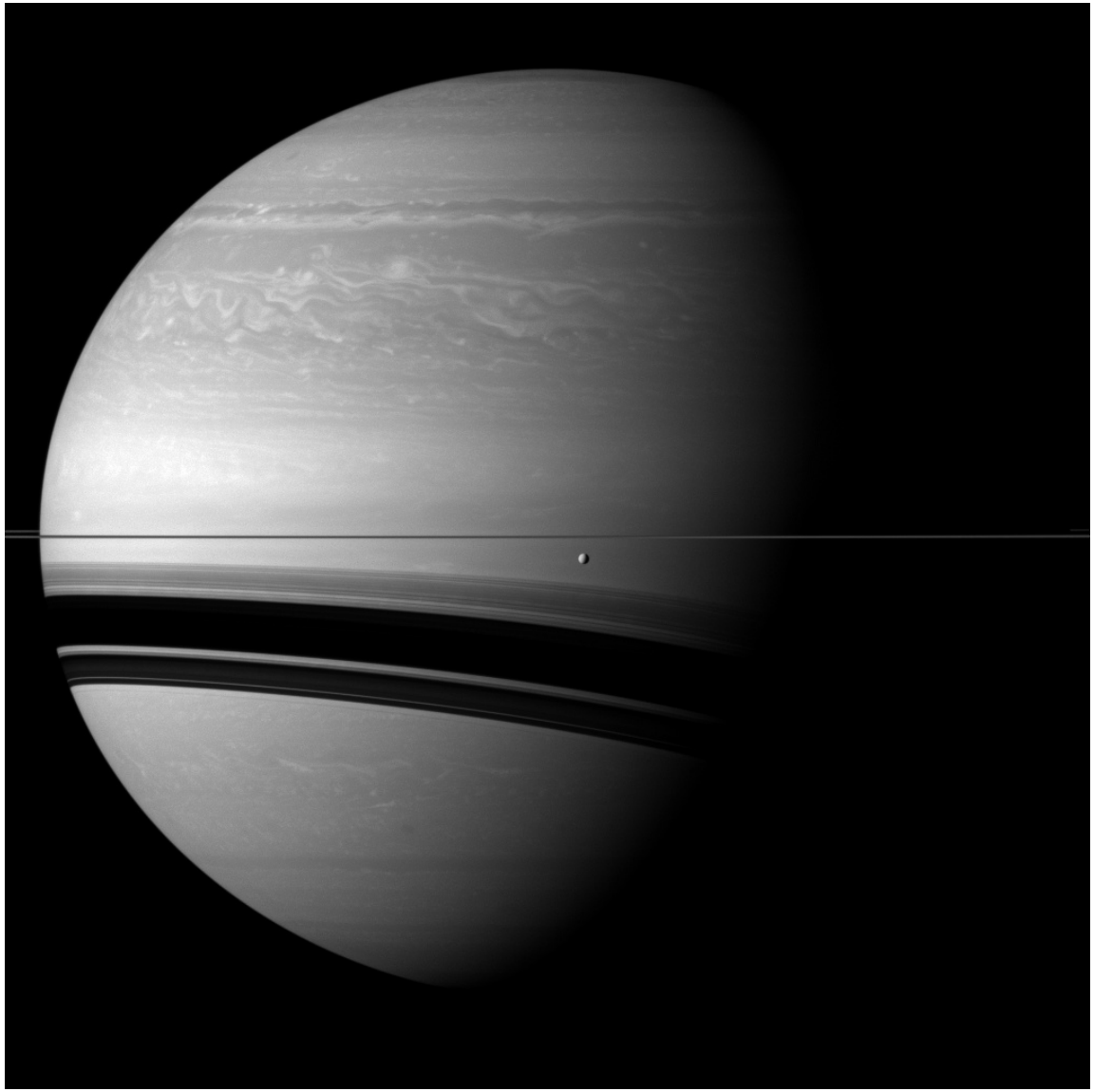
A tale of two time scales

- Atomic time – underlying unit is frequency (*Hz*)
- Civil time – unit is phase (*time of day*)

We should embrace this as a lovely fact of life!

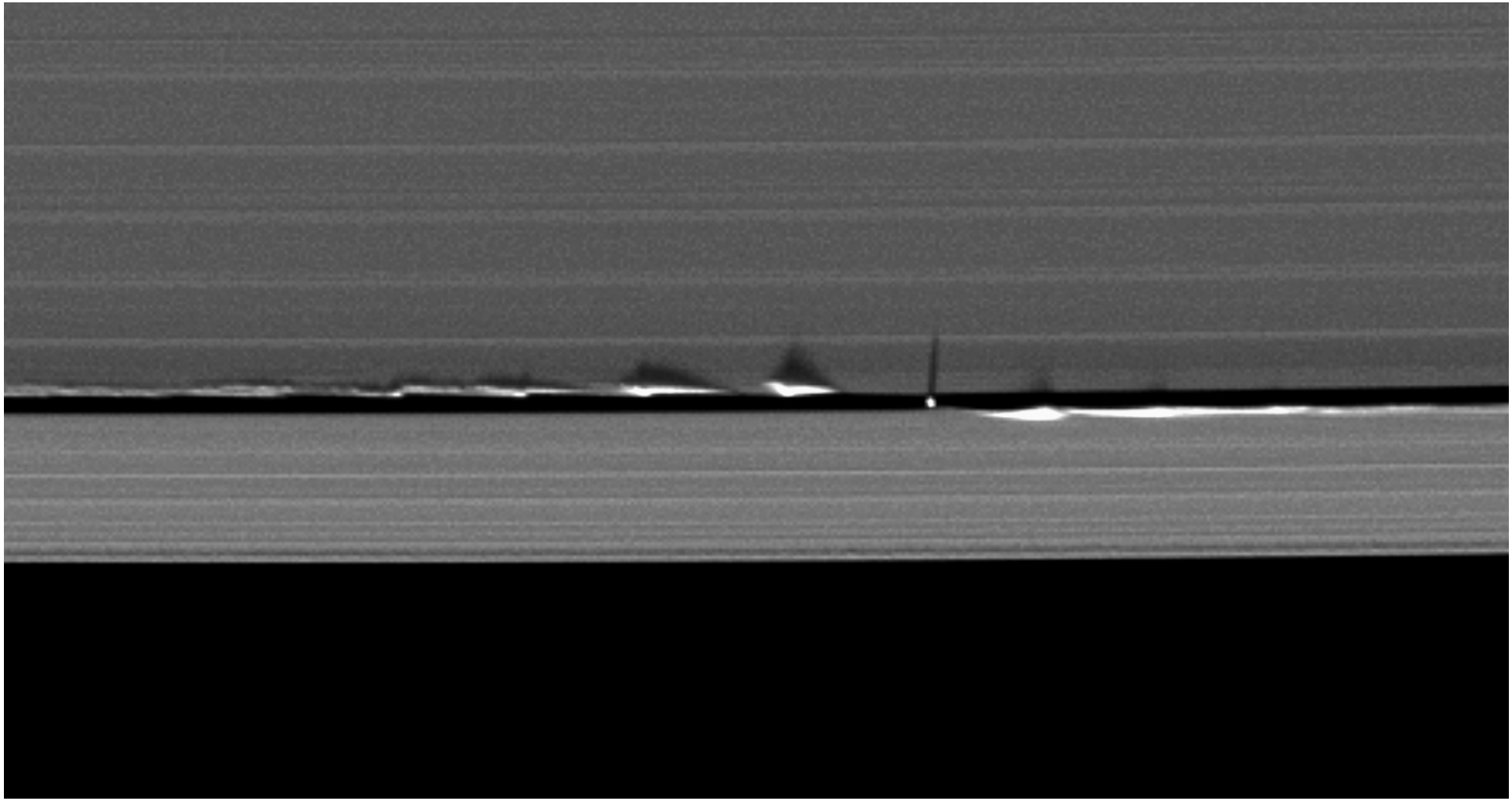
It was the best of times…and it was the best of times

Where else are phase and frequency seen?

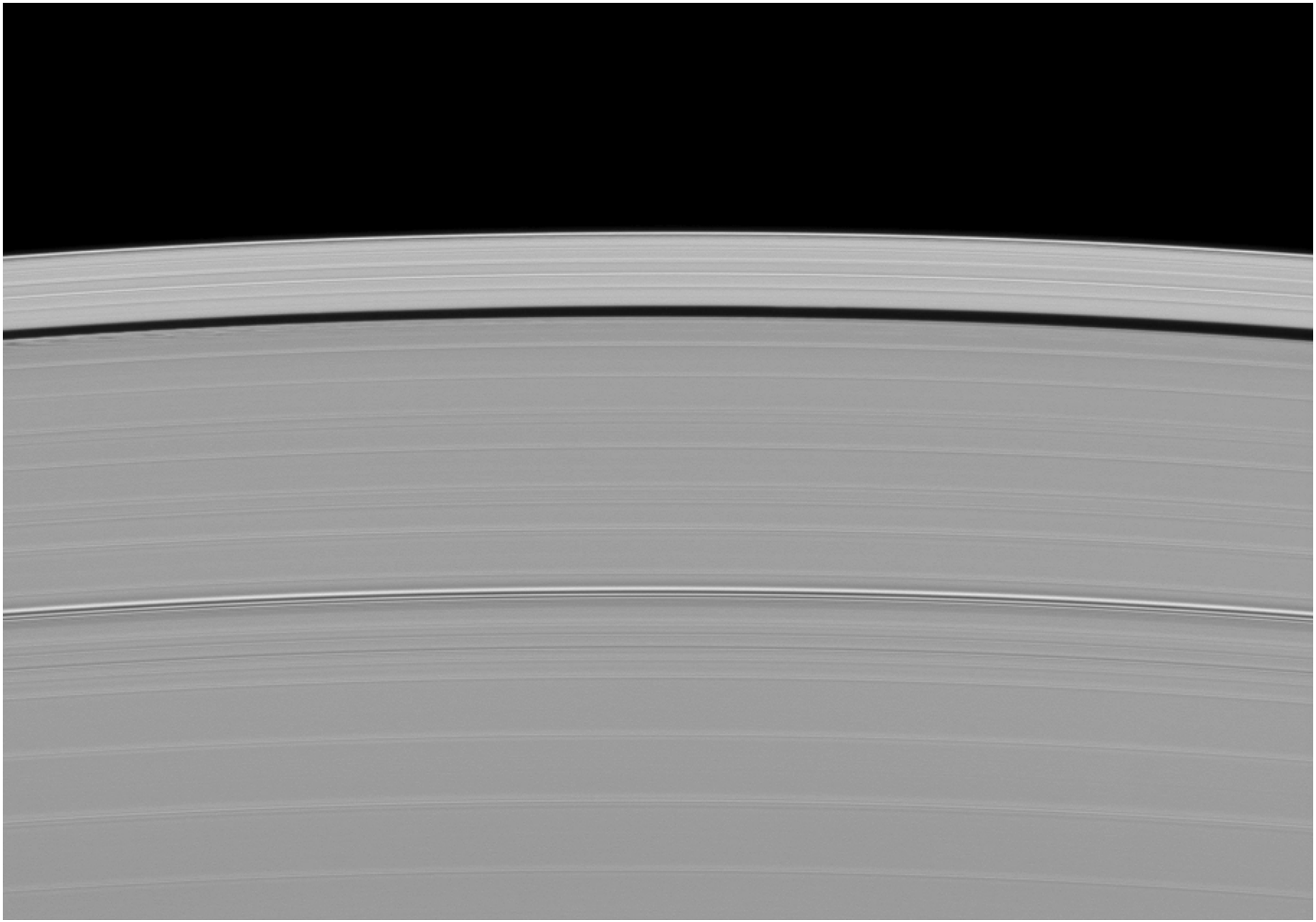


Next 3 slides Cassini Imaging Team

Daphnis and the Keeler gap



Saturn near its equinox

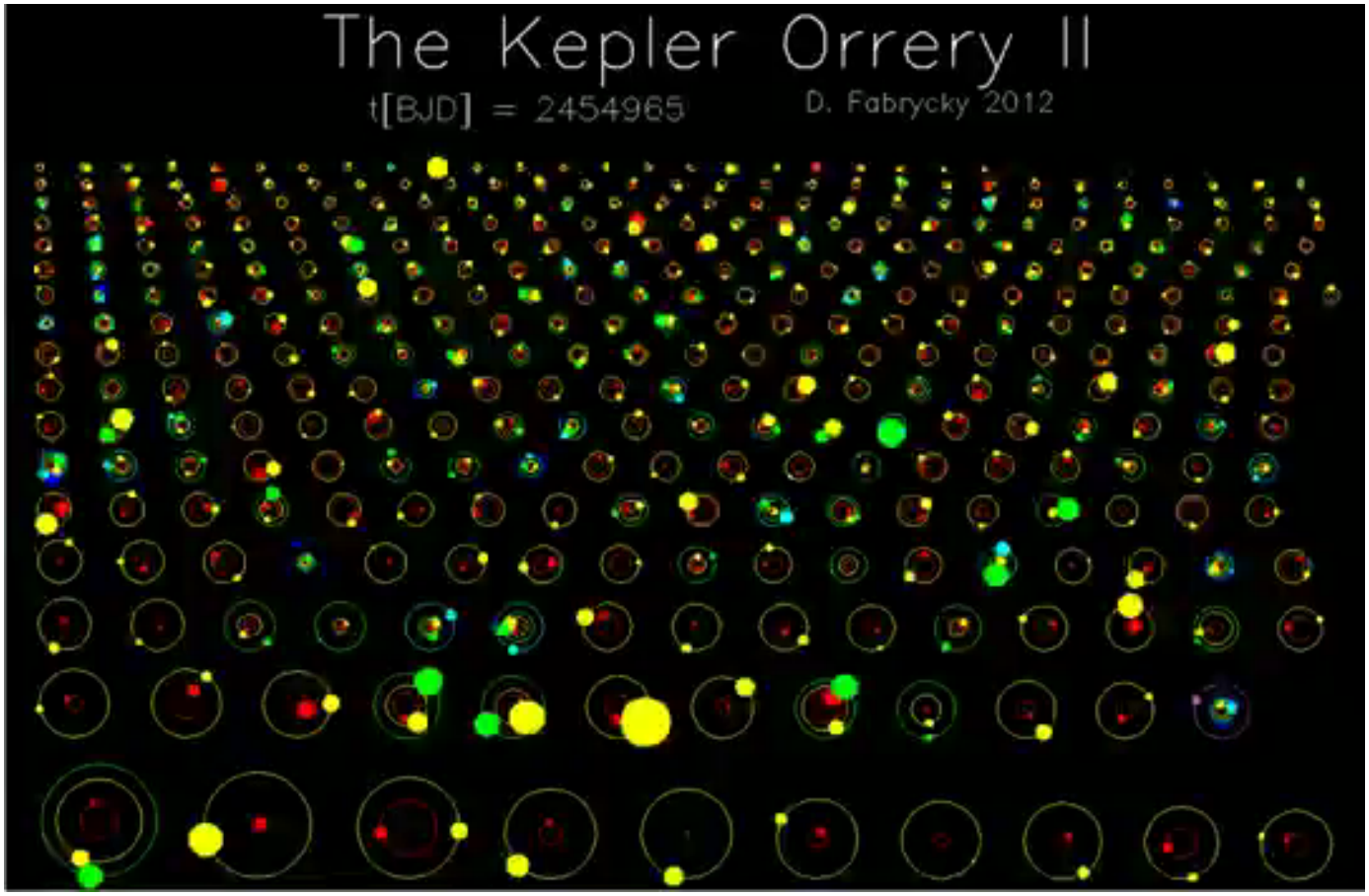


(movie of Daphis shepherding Saturn's rings, http://saturn.jpl.nasa.gov/video/videodetails/?videoID=189)

Day means “synodic day”

- On shepherd moons like Daphnis that orbit within the rings of Saturn
  - The Keeler Gap is about the width of the English Channel and the perpendicular waves preceding and following Daphnis about 5-10x the height of the White Cliffs of Dover – none of this affects the meaning of the word day
- It is the only coherent meaning for “Day”…

Applies to exoplanets, too!



http://kepler.nasa.gov/multimedia/animations/?ImageID=219

(movie of exoplanets discovered by the Kepler mission)

End

Civil Time must mimic Solar Time

...the question is how closely

A: Noon can't become midnight (*Debate*)

B: noticeable amplitude: ~ *hour*

C: time-span over which unacceptable: ~ *decade*

D: divide: max daily error (clock – Earth):

= one hour / one decade
= 3600 s / 3653 days
< 1 s/day

http://futureofutc.org