Using UTC to Determine the Earth's Rotation Angle

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Reference Systems and Frames

- Reference Systems
  - Origin
  - Specified axes
  - Conventional procedures and constants

- Reference Frames
  - Realization
  - List of coordinates and motions
    - Angular
    - Cartesian

Celestial

- **Barycentric**
  - z-axes related to rotational axis
  - x-axes in equatorial plane directed toward a fiducial point
  - third axes complete right-handed orthogonal system

- **Geocentric Celestial Reference System**
  - geocentric space-time coordinates
  - kinematically non-rotating with respect to BCRS.

- **International Celestial Reference System**
  - Idealized system
  - Kinematically non-rotating
  - Orientation
    - close to previous systems
    - independent of epoch,

- **International Celestial Reference *Frame***
  - *s*et of extragalactic objects
  - Successive revisions minimize rotation from original orientation.
  - Angular coordinates of optical stars, consistent with that frame, are provided by the Hipparcos Catalogue

Terrestrial

- **Geocentric**
  - **z-axes fixed to Earth's crust**
  - **x-axes in equatorial plane directed toward a fiducial point**
  - **third axes complete right-handed orthogonal system**

- **Geocentric Terrestrial Reference System**
  - **co-rotating with Earth**

- **International Terrestrial Reference System**
  - **aligned close to mean equator of 1900 and Greenwich meridian,**
  - **continuity with previous terrestrial systems.**
    - **independent of epoch,**

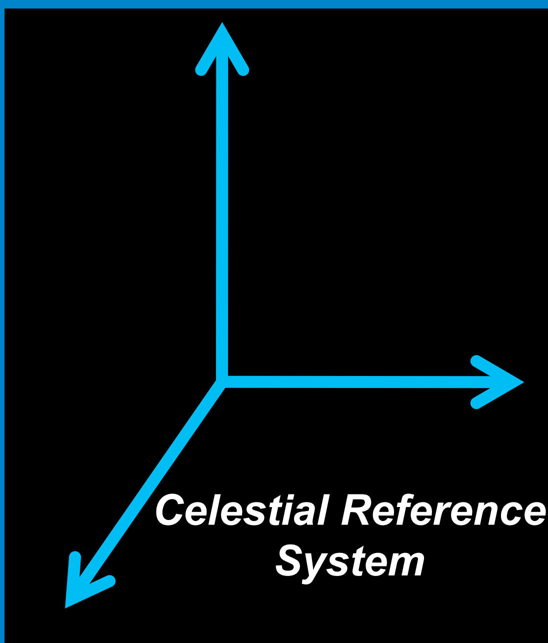
- **International Terrestrial Reference *Frame***
  - **instantaneous coordinates (and velocities) of reference points**
  - **aligned closely to previous terrestrial systems for continuity.**

Intermediate Reference Systems

- **Celestial**
  - **Geocentric**
  - **Related to GCRS by time-dependent rotation (precession-nutation)**
  - **Defined by equator of Celestial Intermediate Pole (CIP) and Celestial Intermediate Origin (CIO) on a specific date**

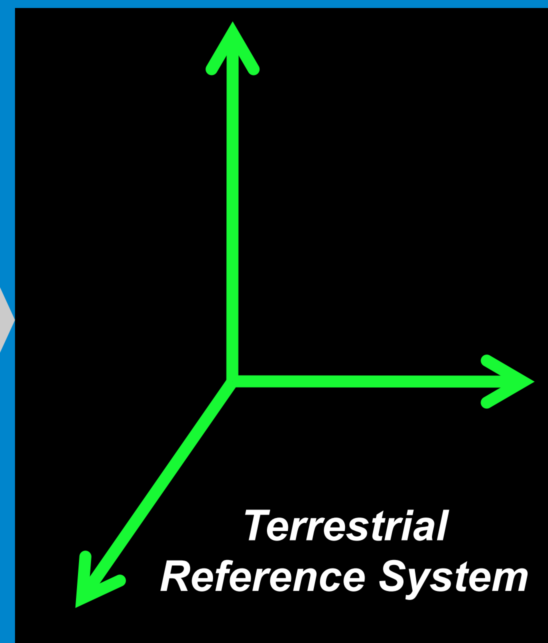
- **Terrestrial**
  - **Geocentric**
  - **Related to ITRS by polar motion**
  - **Related to the Celestial Intermediate Reference System by Earth Rotation Angle (ERA) around the CIP**
  - **Defined by equator of Celestial Intermediate Pole (CIP) and Terrestrial Intermediate Origin (TIO)**

Transforming between Reference Systems



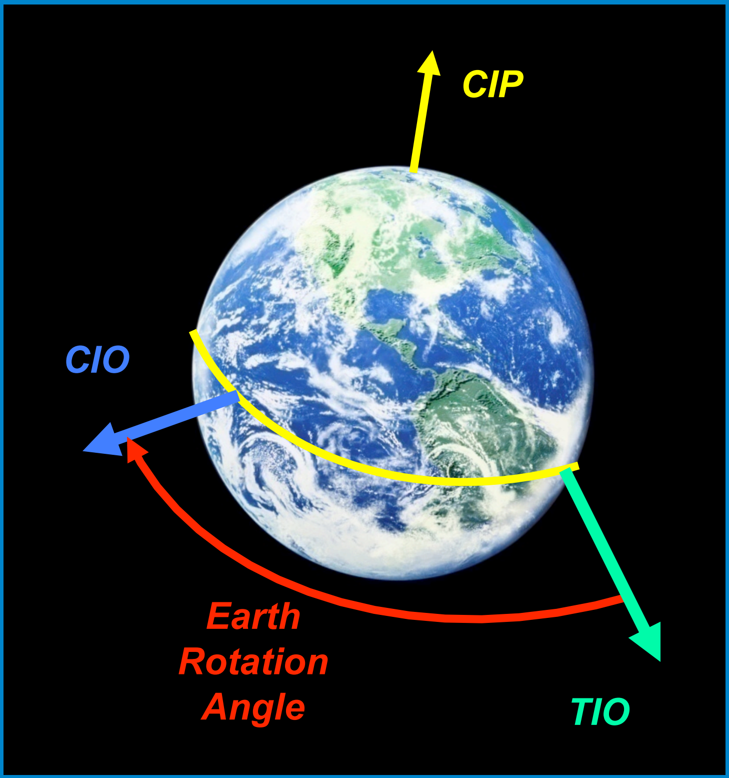
REQUIRES

1. Precession-Nutation Model
2. Polar Motion (x, y)
3. Earth tides model
4. Earth Rotation Angle



Earth Rotation Angle (ERA):

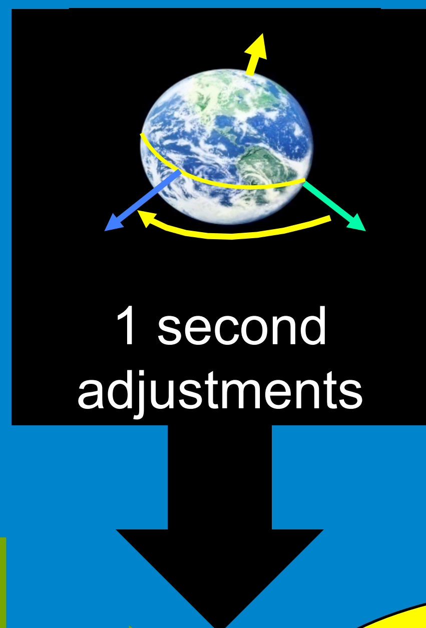
- Angle measured along the intermediate equator of the Celestial Intermediate Pole (CIP) between the Terrestrial Intermediate Origin (TIO) and the Celestial Intermediate Origin (CIO).
- Related to UT1 by a conventionally adopted expression in which ERA is a linear function of UT1
- UT1 is the modern name for historical astronomical time scales, based on the Earth's rotation including Mean Solar Time, Greenwich Mean Time, Universal Time UT

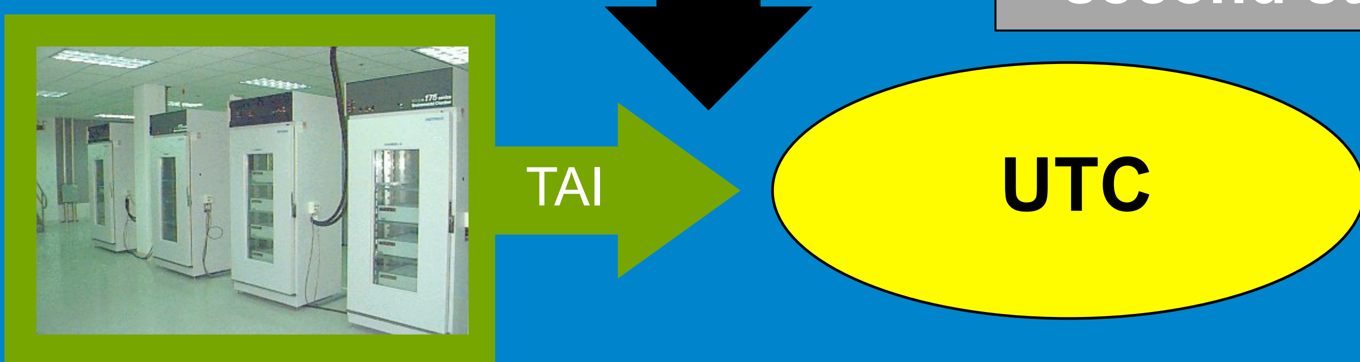


- Variations in the rotational speed of the Earth and the consequent variations in the Earth's rotation angle are conveniently represented by the quantity UT1-UTC (in time units).

Coordinated Universal Time (UTC)

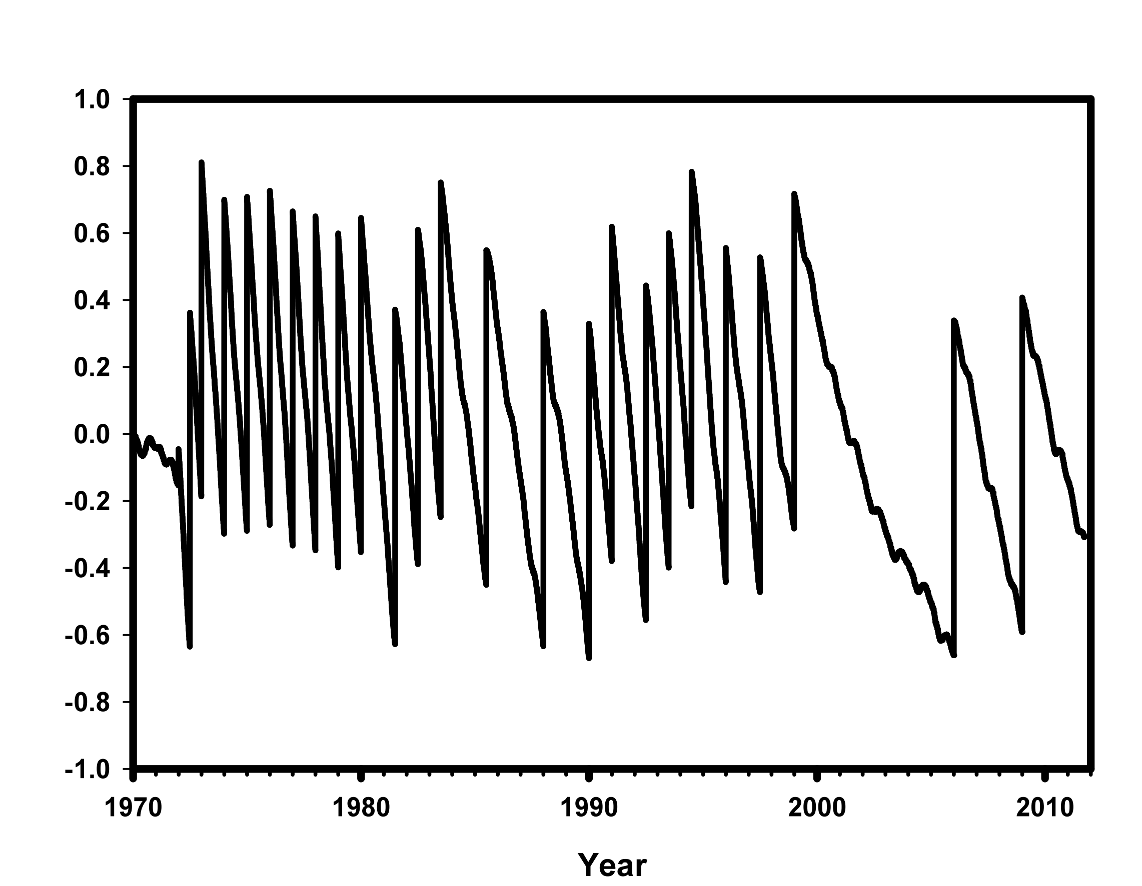
- From 1961 to 1972 UTC contained both rate changes and steps to maintain agreement with UT2 within about 0.1 s
- After 1972 only one-second steps





UT1-UTC

- **Dominant motions**
  - **Trend**
  - **Decadal**
  - **Annual/semiannual**
  - **Tidal**
- **Other smaller amplitude motions**
- **Causes of UT1-UTC**
  - **Tidal deceleration**
  - **Internal changes in inertia tensor**
  - **Atmosphere (winds)**
  - **Solid Earth tides**



Transforming Coordinates

Reference pole motion in space (model)

Rotation angle (observed/predicted)

Reference pole motion on Earth (observed/predicted)

$$[CRS] = PN(t)\, R(t)\, W(t)\, [TRS]$$

Space "Coordinate"

Earth "Coordinate"

Earth Rotation Angle

Using Greenwich Sidereal Time (Legacy)

$$R(t) = R_3(-GST)$$

$$= \begin{pmatrix} \cos GST & -\sin GST & 0 \\ \sin GST & \cos GST & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Equation of equinoxes = $\Delta\psi \cos \varepsilon_A$

Tidal Corrections

IAU Recommended

$$R(t) = R_3(-\theta)$$

$$= \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Tidal Corrections

Software at http://maia.usno.navy.mil/ch5subs.html

ERA2000 subroutine produces the Earth rotation angle θ

Greenwich Sidereal Time

- $GST = ERA(UT1) - EO$
- $EO = -0.01450600'' - 4612.15653400''t - 1.391581700''t^2$
  $+ 0.0000004400''t^3 - \Delta\psi \cos\varepsilon_A - P$
- t = (Terrestrial Time (TT) - 2451545.0TT)/36525 and $\Delta\psi \cos\varepsilon_A$ is the classical equation of the equinoxes
- P represents a series of periodic terms given in Table 5.2e of the IERS Conventions (2010).

ERA

$$ERA(T_U) = \theta(T_U)$$

$$= 2\pi\,(0.779\,057\,273\,264\,0 + 1.002\,737\,811\,911\,354\,48\,T_U)$$

$$T_u = (\text{Julian UT1 date} - 2451545.0),$$

Software at http://maia.usno.navy.mil/ch5subs.html

ERA2000 subroutine produces the Earth rotation angle θ

UT1

- **Both methods require UT1**
- **Computed from**

$$UT1 = UTC + (UT1 - UTC)$$

TT

= International Atomic Time (TAI) + 32.184s

= UTC

+ [(TAI – UTC) + 32.184s]

FROM			TO			TAI-UTC				
1961	Jan.	1	1961	Aug.	1	1.4228180s	+	(MJD-37300)	x	0.001296s
	Aug.	1	1962	Jan.	1	1.3728180s	+	(MJD-37300)	x	0.001296s
1962	Jan.	1	1963	Nov.	1	1.8458580s	+	(MJD-37665)	x	0.0011232s
1963	Nov.	1	1964	Jan.	1	1.9458580s	+	(MJD-37665)	x	0.0011232s
1964	Jan.	1		April	1	3.241300s	+	(MJD-38761)	x	0.001296s
	April	1		Sept.	1	3.341300s	+	(MJD-38761)	x	0.001296s
	Sept.	1	1965	Jan.	1	3.441300s	+	(MJD-38761)	x	0.001296s
1965	Jan.	1		March	1	3.541300s	+	(MJD-38761)	x	0.001296s
	March	1		Jul.	1	3.641300s	+	(MJD-38761)	x	0.001296s
	Jul.	1		Sept.	1	3.741300s	+	(MJD-38761)	x	0.001296s
	Sept.	1	1966	Jan.	1	3.841300s	+	(MJD-38761)	x	0.001296s
1966	Jan.	1	1968	Feb.	1	4.3131700s	+	(MJD-39126)	x	0.002592s
1968	Feb.	1	1972	Jan.	1	4.2131700s	+	(MJD-39126)	x	0.002592s
1972	Jan.	1		Jul.	1	10s				
	Jul.	1	1973	Jan.	1	11s				
1973	Jan.	1	1974	Jan.	1	12s				
1974	Jan.	1	1975	Jan.	1	13s				
1975	Jan.	1	1976	Jan.	1	14s				
1976	Jan.	1	1977	Jan.	1	15s				
1977	Jan.	1	1978	Jan.	1	16s				
1978	Jan.	1	1979	Jan.	1	17s				
1979	Jan.	1	1980	Jan.	1	18s				
1980	Jan.	1	1981	Jul.	1	19s				
1981	Jul.	1	1982	Jul.	1	20s				
1982	Jul.	1	1983	Jul.	1	21s				
1983	Jul.	1	1985	Jul.	1	22s				
1985	Jul.	1	1988	Jan.	1	23s				
1988	Jan.	1	1990	Jan.	1	24s				
1990	Jan.	1	1991	Jan.	1	25s				
1991	Jan.	1	1992	Jul.	1	26s				
1992	Jul.	-1	1993	Jul	1	27s				
1993	Jul.	1	1994	Jul.	1	28s				
1994	Jul.	1	1996	Jan.	1	29s				
1996	Jan.	1	1997	Jul.	1	30s				
1997	Jul.	-1	1999	Jan.	1	31s				
1999	Jan.	-1	2006	Jan.	1	32s				
2006	Jan.	-1	2009	Jan.	1	33s				
2009	Jan.	-1				34s				

Sources of UT1

- **<u>UTC</u>**
  - **Leap seconds imply UT1 = UTC**
  - **Error < 0.9s**
- **<u>IERS Observed</u>**
  - **UT1-UTC with error < 10 μs**
- **<u>IERS Predicted</u>**
  - **Based on**
    - **Observations**
    - **Models**
  - **Sort term**
  - **Long term**
- **<u>GPS III</u>**

Diurnal and semi-diurnal tidal corrections to UT1 are available

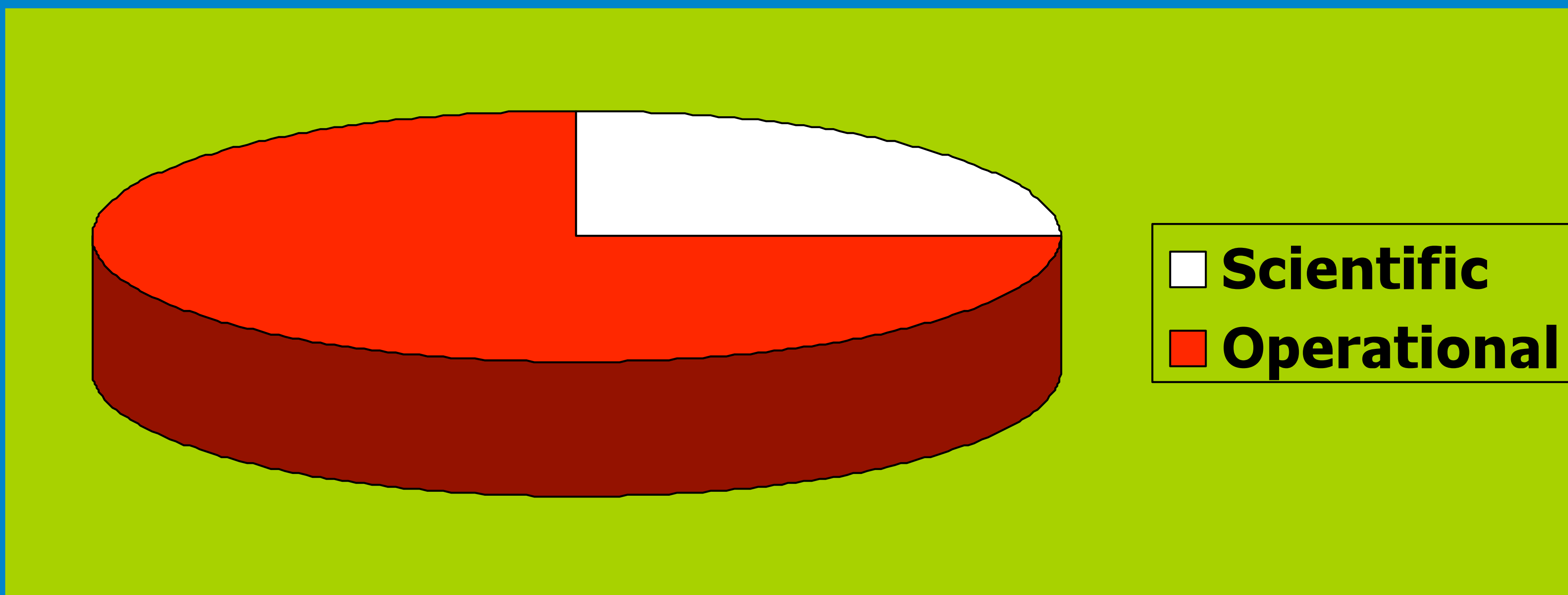
IERS (International Earth Rotation and Reference System Service)

- **Established as International Earth Rotation Service in 1987 by International Astronomical Union and International Union of Geodesy and Geophysics**
- **In 2003 renamed to International Earth Rotation and Reference Systems Service.**
- **Serves astronomical, geodetic and geophysical communities by providing:**
  - **International Celestial Reference System (ICRS) and its realization, the International Celestial Reference Frame (ICRF).**
  - **International Terrestrial Reference System (ITRS) and its realization, the International Terrestrial Reference Frame (ITRF).**
  - **Earth orientation parameters required to transform between the ICRF and the ITRF and for research.**
  - **Geophysical data to interpret time/space variations in the ICRF, ITRF or earth orientation parameters, and model such variations.**
  - **Standards, constants and models (*i.e.*, conventions) encouraging international adherence.**

IERS Bulletin A

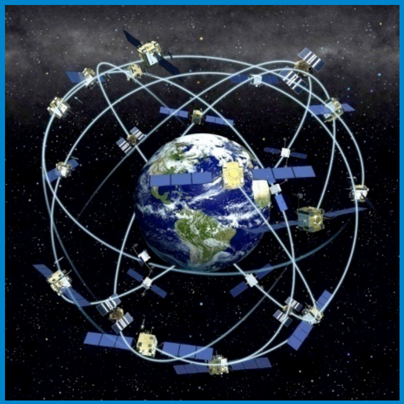
			MJD	x	error	y	error	UT1-UTC	error
				"	"	"	"	s	s
8	1	4	54469	-0.08556	0.00009	0.26378	0.00010	-0.275953	0.000009
8	1	5	54470	-0.08795	0.00009	0.26552	0.00009	-0.276615	0.000011
8	1	6	54471	-0.09033	0.00009	0.26714	0.00010	-0.277208	0.000012
8	1	7	54472	-0.09261	0.00009	0.26880	0.00010	-0.277746	0.000012
8	1	8	54473	-0.09446	0.00009	0.27059	0.00010	-0.278265	0.000013
8	1	9	54474	-0.09605	0.00009	0.27260	0.00010	-0.278838	0.000048
8	1	10	54475	-0.09757	0.00009	0.27480	0.00010	-0.279595	0.000057

Who are the Users?



Uses

Positioning
and
Navigation

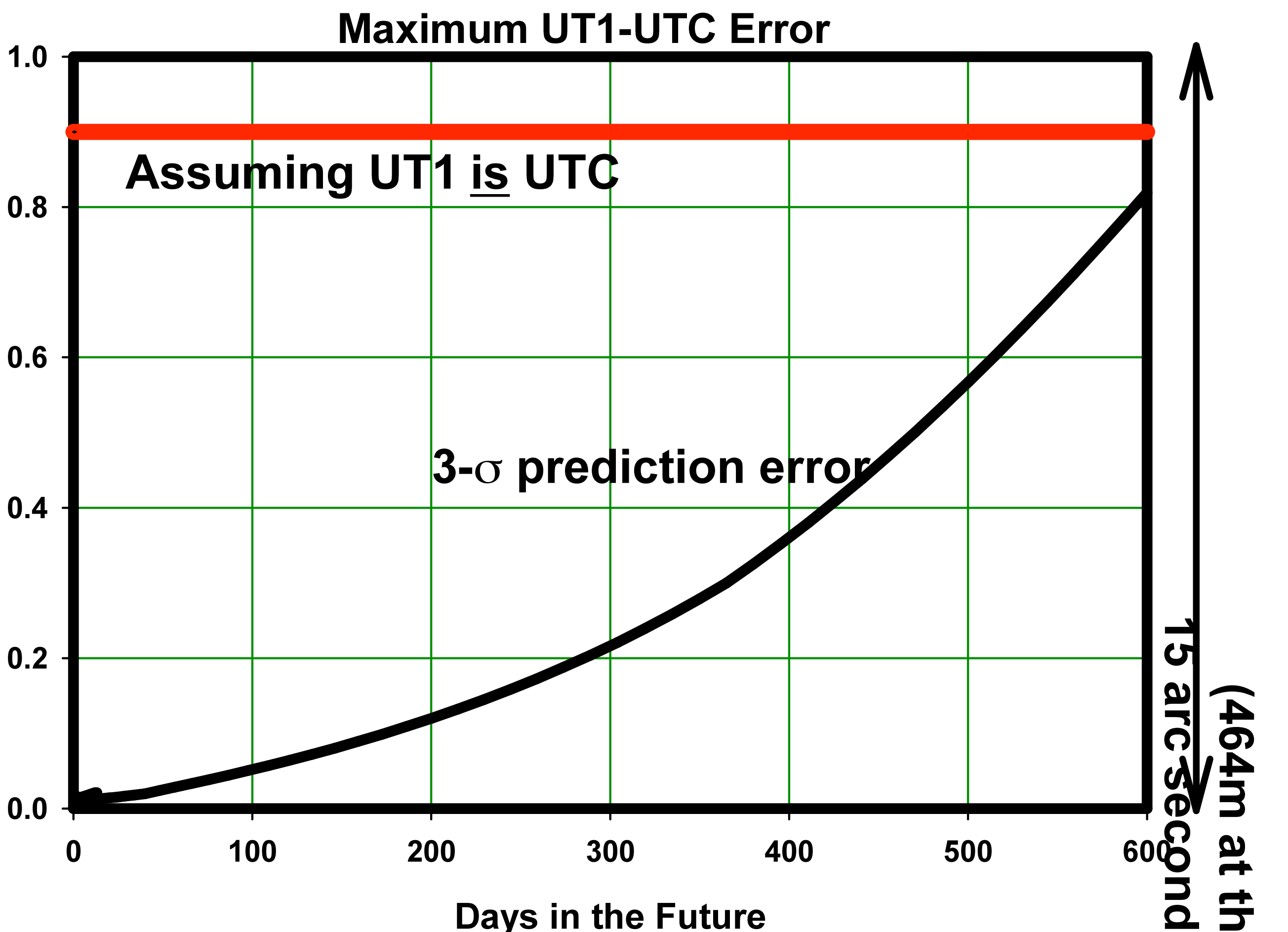


Pointing

Timekeeping

Research

UT1-UTC Error



UT1 since 1970…

1970	**2011**
• UT1 required for celestial navigation	• Diminished use of celestial Navigation
• UT1 observation accuracy ~1 ms	• UT1 observation accuracy ~10 μs
• No predictions of UT1 available	• Near real time UT1 and predictions available
• Printed bulletins only source	• Near real time electronic access

Conclusion

<u>"Legacy"</u>

R(t) computed from

Greenwich Sidereal Time

<u>IAU Recommended</u>

R(t) computed from

Earth Rotation Angle

Either method requires a value for UT1 by computing

UT1=UTC + (UT1-UTC)

UT1-UTC available from IERS/USNO

- **Full Accuracy**
- **Fundamental solution**

OR

Ignore UT1-UTC

- **Errors as large as 13.5 sec. of arc.**

<u>**Software** at http://maia.usno.navy.mil/ch5subs.html</u>

ERA2000 subroutine produces the Earth rotation angle θ