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USING UTC TO DETERMINE THE EARTH'S ROTATION ANGLE

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The Earth's rotation angle is a critical component of the suite of five Earth orientation parameters used to transform between terrestrial and celestial reference systems. This angle is defined mathematically using an adopted conventional relationship between UT1 and the mathematical quantity known as "Earth Rotation Angle" (ERA). For practical purposes, then, UT1–UTC provides a convenient means to obtain UT1, knowing UTC, and thus the ERA. Because the Earth's rotational speed is variable, it is not practical to model UT1 as a function of time with the accuracy needed for many applications. Consequently astronomical and geodetic institutions from around the world share observations of the Earth's rotation angle and these data are then used to provide users the latest observations of UT1–UTC as well as predicted estimates with accuracy that depends on the prediction interval. This process can provide users with daily updates of UT1–UTC with accuracy of the order of tens of microseconds and predictions with accuracy better than 1 millisecond up to ten days in advance. The International Earth Rotation and Reference Systems Service (IERS) was established in 1987 by the International Astronomical Union and the International Union of Geodesy and Geophysics to provide this information operationally. In addition to the services routinely providing UT1 with sub-millisecond accuracy, UTC is currently adjusted to keep |UT1–UTC| < 0.9 seconds, and this definition provides a means to access UT1 automatically with accuracy of the order of one second. Should UTC be defined without the restriction keeping |UT1–UTC| < 0.9 seconds, the low accuracy estimate of UT1 (±1 second) would no longer be assured. However the existing national and international services can be expected to provide the current products as they do now via paper bulletin and electronic means. It is assumed that the accuracy of those products will always reflect the state of the art. In the future, high-speed transfer of high-quality observational astronomical, meteorological, oceanic and geophysical data promise to decrease the latency of the observations and provide UT1–UTC at sub-daily intervals with increasingly improving accuracy. In addition to the current means of distribution, increasing access to electronic communication services has the potential to provide near real-time, state of the art UT1–UTC to users when and wherever it is needed. If there were sufficient demand, we might even envision a UT1–UTC application being made available for future hand-held devices.

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INTRODUCTORY BACKGROUND

It is important to distinguish between reference systems and reference frames when discussing the use of Earth orientation parameters. Reference systems, either terrestrial or celestial, have an origin, specified directions of three fundamental dimensional axes, and a set of conventional models, procedures, and constants used in the actual realization of the system. A reference frame, on the other hand, is the realization of that system through a list of coordinates, either angular or Cartesian.

Celestial reference systems generally have their origins at the barycenter of the solar system, and their polar axes (z-axes) related in some way to the rotational axis of the Earth. The second axis (x-axis) then lies in the equatorial plane perpendicular to the z-axis and is directed toward a fiducial point in that plane. The third axis is chosen to complete a right-handed orthogonal system. In astronomical applications the International Celestial Reference System (ICRS) is the idealized barycentric coordinate system to which celestial positions are referred. It is kinematically non-rotating with respect to distant extragalactic objects. It was aligned close to previous astronomical reference systems for continuity. Its orientation is independent of epoch, ecliptic or equator and is realized by a list of adopted coordinates of extragalactic sources. The Geocentric Celestial Reference System (GCRS) is a system of geocentric space-time coordinates defined such that the transformation between BCRS and GCRS spatial coordinates contains no rotation component, so that GCRS is kinematically non-rotating with respect to BCRS. The spatial orientation of the GCRS is derived from that of the BCRS. The International Celestial Reference Frame (ICRF), then, is a set of extragalactic objects whose adopted positions and uncertainties realize the ICRS axes.[1] It is also the name of the radio catalog listing the directions to defining sources. Successive revisions of the ICRF are intended to minimize rotation from its original orientation. Angular coordinates of optical stars, consistent with that frame, are provided by the Hipparcos Catalogue.[2]

Terrestrial reference systems generally have their origins at the center of mass of the Earth with their polar axes related to the direction of an axis fixed with respect to the Earth's crust. The origin of longitudes in the equatorial plane provides the second direction. Again, a third axis is chosen to complete a right-handed orthogonal system. The Geocentric Terrestrial Reference System (GTRS) is a system of geocentric space-time coordinates co-rotating with the Earth. The International Terrestrial Reference System (ITRS) is a specific GTRS for which the co-rotation condition is defined as no residual rotation with regard to the Earth's surface, and the geocenter is understood as the center of mass of the whole Earth system, including oceans and atmosphere. It was aligned close to the mean equator of 1900 and the Greenwich meridian, for continuity with previous terrestrial reference systems. The International Terrestrial Reference Frame (ITRF), is a realization of ITRS by a set of instantaneous coordinates (and velocities) of reference points distributed on the topographic surface of the Earth (mainly space geodetic stations and related markers).[3] Its initial orientation of the ITRF is aligned closely to previous terrestrial systems for continuity.

The Celestial Intermediate Reference System (CIRS) is a geocentric reference system related to the GCRS by a time-dependent rotation taking into account precession-nutation. It is defined by the intermediate equator of the Celestial Intermediate Pole (CIP) and the Celestial Intermediate Origin (CIO) on a specific date. The CIP is a geocentric equatorial pole defined as being the intermediate pole in the transformation from the GCRS to the ITRS, separating nutation from polar motion. Its GCRS orientation results from the part of precession-nutation with periods greater than 2 days, the retrograde diurnal part of polar motion (including the free core nutation, FCN) and a reference frame bias. Its ITRS orientation is comprised of the part of polar motion which is outside the retrograde diurnal band in the ITRS and the motion in the ITRS corresponding to nutational motions with periods less than 2 days. The motion of the CIP is realized by the IAU precession-nutation plus small time-dependent corrections called "celestial pole offsets." The CIO is the origin for right ascension on the intermediate equator in the CIRS. It is the non-rotating origin in the GCRS originally set close to the GCRS meridian and throughout 1900-2100 stays within 0.1 arc seconds of this alignment. The CIO was located on the CIP equator of J2000.0 at a direction 2.012 milli-arcseconds (mas) from the ICRS prime meridian at right ascension 0h 0m 0s.000 134 16 in the ICRS. As the true equator moves in space, the path of the CIO in space is such that the point has no instantaneous east-west velocity along the true equator. In contrast, the equinox defined by the intersection of the equator and the plane of the ecliptic has instantaneous velocity along the equator.

The Terrestrial Intermediate Reference System (TIRS) is a geocentric reference system defined by the intermediate equator of the CIP and the Terrestrial Intermediate Origin (TIO). It is related to the ITRS by polar motion and the TIO locator. It is related to the Celestial Intermediate Reference System by the Earth Rotation Angle (ERA) around the CIP that realizes the common z-axis of the two systems. The TIO is the origin of longitude in the ITRS. It is the non-rotating origin in the ITRS that was originally set at the ITRF origin of longitude and throughout 1900-2100 stays within 0.000 1″ of the ITRF zero meridian.

The terrestrial system rotates in the celestial system and its orientation in that system is affected by precession, nutation, polar motion and variations in the Earth's rotational speed. The fact that the Earth is not strictly a rigid body means that non-rigid body effects need to be considered in models of the Earth's rotational motions, and because the Earth's core experiences a free wobble with respect to the mantle, existing geophysical models of nutation may not account for all of the observed motions. Further, motions caused by redistribution of mass in the Earth, its oceans and atmosphere, along with relatively high-frequency variations in global meteorology and hydrology may also need to be taken into account.

With the introduction of a new reference system in the 1992–2004 period, the CIO replaced the moving vernal equinox; the TIO replaced the Greenwich Meridian; and the Earth Rotation Angle (ERA) replaced the Greenwich Sidereal Time. The alternative system based on the equinox, mean and true positions, and the Greenwich Mean Sidereal Time is still supported and when properly applied can provide equivalent accuracies.[4]

EARTH ORIENTATION PARAMETERS

The transformation between celestial and terrestrial frames is specified by five angles called Earth orientation parameters. The rigorous details are outlined in the publications of the International Earth Rotation and Reference Systems Service (IERS), specifically in the *IERS Conventions (2010)* and its updates, which are available electronically at http://tai.bipm.org/iers/conv2010/conv2010.html.[5] Three would be sufficient, but five angles are used in order to describe the physical processes involved and to make the transformations easier to apply. Two angles are used to model the changing direction of the CIP due to the precession and nutation of the Earth. These phenomena are driven by the gravitational attraction of the solar system bodies, principally the Sun and the Moon, on the non-spherical Earth. Precession refers to the aperiodic portion of the motion and nutation refers to the periodic portion. Both motions depend on the positions of the solar system bodies and the internal structure of the Earth, but they can be modeled mathematically with reasonable accuracy.

Two more angles are used to describe the motion of the CIP with respect to the Earth's crust. This phenomenon called "polar motion" is driven by geophysical and meteorological variations within the Earth and its atmosphere. Polar motion is difficult to model because the forces driving the motion are difficult to predict. As a result these angles must be observed astronomically and made available to users operationally.

The last of the five angles characterizes the rotation angle of the Earth and is described by the time difference UT1–UTC. UT1 is a measure of the Earth's rotation angle expressed in time units and treated conventionally as an astronomical time scale defined by the rotation of the Earth with respect to the Sun. It is expressed in time units rather than in degrees, minutes, and seconds of arc because of its historical use in providing a standard international time scale. In practice, UT1 was defined until 1 January 2003 by means of a conventional formula (Aoki, *et al.*, 1982).[6] It is now defined as being linearly proportional to the ERA, and the transformation between the ITRS and GCRS is specified using the ERA. The ERA is the angle measured along the intermediate equator of the CIP between the TIO and the CIO, positively in the retrograde direction and increasing linearly for an ideal, uniformly rotating Earth. It is related to UT1 by a conventionally adopted expression in which ERA is a linear function of UT1.

$$ERA(T_U) = \theta(T_U) = 2\pi(0.779\,057\,273\,264\,0 + 1.002\,737\,811\,911\,354\,48 T_U), \qquad (1)$$

where T_U = (Julian UT1 date - 2451545.0), and UT1 = UTC + (UT1–UTC). Its time derivative is the Earth's angular velocity (Figure 1).

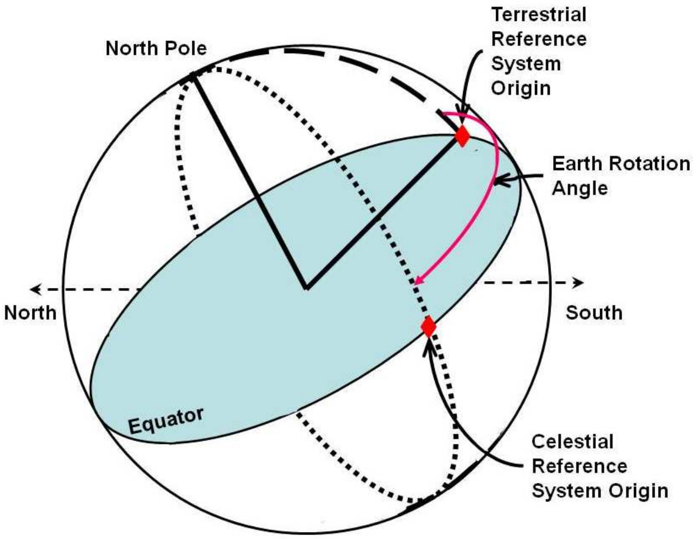

Figure 1. The Earth Rotation Angle.

UT1 is determined operationally by astronomical observations (currently from VLBI observations of the diurnal motions of distant radio sources and other observations), and was originally defined as a kind of time determined by the rotation of the Earth. It is obtained in practice as a time difference from the international time standard, Coordinated Universal Time (UTC), by using the quantity UT1–UTC. The use of the Earth as a timekeeping device became problematic, when it became apparent that its rotational speed was variable and the need for a uniform time scale began to grow. Principal variations in the rotation speed of the Earth include a constant deceleration due to tidal deceleration and de-glaciation, decadal variations due to changes in the internal distribution of the Earth's mass, largely seasonal, meteorologically driven variations and tidally driven periodic variations. As with polar motion, UT1–UTC is difficult to model and predict, and must be observed astronomically and reported to users routinely.

Greenwich Sidereal Time (GST) is an angle that is the sum of the ERA and the angular distance between the CIO and a conventional equinox along the moving equator. This distance is called the Equation of Origins (EO) which is the CIO based right ascension of the equinox along the moving equator and corresponds to the accumulated precession and nutation in right ascension from the epoch of reference to the current date. This relationship can be written as

$$GST = ERA(UT1) - EO, \tag{2}$$

where EO is given by

$$\begin{aligned} EO = &-0.01450600'' - 4612.15653400''t - 1.391581700''t^2 + 0.0000004400''t^3 \\ &- \Delta\psi \cos\varepsilon_A - P, \end{aligned} \tag{3}$$

and t = (Terrestrial Time (TT) – 2451545.0 TT) / 36525, $\Delta\psi \cos\varepsilon_A$ is the classical equation of the equinoxes, and P represents a series of periodic terms given in Table 5.2e of the *IERS Conventions (2010)*.[5]

RELATING THE ITRS TO THE GCRS

The transformation to be used to relate the ITRS to the GCRS at the date t of the observation can be written as:

$$[\mathbf{GCRS}] = \mathbf{Q}(t)\,\mathbf{R}(t)\,\mathbf{W}(t)\,[\mathbf{ITRS}], \tag{4}$$

where $\mathbf{Q}(t)$, $\mathbf{R}(t)$ and $\mathbf{W}(t)$ are the transformation matrices arising from the motion of the celestial pole in the celestial reference system, from the rotation of the Earth around the axis associated with the pole, and from polar motion respectively. The details are provided in the *IERS Conventions (2010)*.[5] UT1 is only involved in $\mathbf{R}(t)$. The CIO based transformation matrix arising from the rotation of the Earth around the axis of the CIP can be expressed as $\mathbf{R}(t) = R_3(-ERA)$ or

$$\mathbf{R}(t) = \begin{bmatrix} \cos(ERA) & -\sin(ERA) & 0 \\ \sin(ERA) & \cos(ERA) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The equinox based transformation matrix R(t) for Earth rotation transforms from the TIRS to the true equinox and equator of date system using Apparent Greenwich Sidereal Time (*GST*), i.e. the angle between the equinox and the TIO, to represent the Earth's angle of rotation, instead of the ERA, is $\mathbf{R}(t) = R_3(-GST)$ or

$$\mathbf{R}(t) = \begin{bmatrix} \cos(GST) & -\sin(GST) & 0 \\ \sin(GST) & \cos(GST) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

It is then apparent that two kinds of time are involved in forming $\mathbf{R}(t)$, *i.e.* TT and UT1. Both are provided by their relationships to the international time standard, Coordinated Universal Time (UTC). The TT epoch can be determined in relation to a UTC epoch by using

$$\text{TT} = \text{International Atomic Time (TAI)} + 32.184\text{s} = \text{UTC} + [(\text{TAI} - \text{UTC}) + 32.184\text{s}] \tag{7}$$

and the values of TAI-UTC given in Table 1. UT1 can be determined by using

$$\mathrm{UT1} = \mathrm{UTC} + (\mathrm{UT1} - \mathrm{UTC}) \tag{8}$$

and values of UT1–UTC provided by a variety of sources. In cases where the application may permit, some users also may just neglect UT1–UTC by effectively setting it to zero knowing that, through the use of leap seconds, |UT1–UTC| < 0.9s. However for users who require accuracy better than 1 second of time (equivalent to 15 seconds of arc or 464 meters at the Earth's equator) non-zero values of UT1–UTC must be used.

Table 1. Values of TAI-UTC.

FROM			TO			TAI-UTC				
1961	Jan.	1	1961	Aug.	1	1.4228180s	+	(MJD-37300)	x	0.001296s
	Aug.	1	1962	Jan.	1	1.3728180s	+	(MJD-37300)	x	0.001296s
1962	Jan.	1	1963	Nov.	1	1.8458580s	+	(MJD-37665)	x	0.0011232s
1963	Nov.	1	1964	Jan.	1	1.9458580s	+	(MJD-37665)	x	0.0011232s
1964	Jan.	1		April	1	3.241300s	+	(MJD-38761)	x	0.001296s
	April	1		Sept.	1	3.341300s	+	(MJD-38761)	x	0.001296s
	Sept.	1	1965	Jan.	1	3.441300s	+	(MJD-38761)	x	0.001296s
1965	Jan.	1		March	1	3.541300s	+	(MJD-38761)	x	0.001296s
	March	1		Jul.	1	3.641300s	+	(MJD-38761)	x	0.001296s
	Jul.	1		Sept.	1	3.741300s	+	(MJD-38761)	x	0.001296s
	Sept.	1	1966	Jan.	1	3.841300s	+	(MJD-38761)	x	0.001296s
1966	Jan.	1	1968	Feb.	1	4.3131700s	+	(MJD-39126)	x	0.002592s
1968	Feb.	1	1972	Jan.	1	4.2131700s	+	(MJD-39126)	x	0.002592s
1972	Jan.	1		Jul.	1	10s				
	Jul.	1	1973	Jan.	1	11s				
1973	Jan.	1	1974	Jan.	1	12s				
1974	Jan.	1	1975	Jan.	1	13s				
1975	Jan.	1	1976	Jan.	1	14s				
1976	Jan.	1	1977	Jan.	1	15s				
1977	Jan.	1	1978	Jan.	1	16s				
1978	Jan.	1	1979	Jan.	1	17s				
1979	Jan.	1	1980	Jan.	1	18s				
1980	Jan.	1	1981	Jul.	1	19s				
1981	Jul.	1	1982	Jul.	1	20s				
1982	Jul.	1	1983	Jul.	1	21s				
1983	Jul.	1	1985	Jul.	1	22s				
1985	Jul.	1	1988	Jan.	1	23s				
1988	Jan.	1	1990	Jan.	1	24s				
1990	Jan.	1	1991	Jan.	1	25s				
1991	Jan.	1	1992	Jul.	1	26s				
1992	Jul.	1	1993	Jul	1	27s				
1993	Jul.	1	1994	Jul.	1	28s				
1994	Jul.	1	1996	Jan.	1	29s				
1996	Jan.	1	1997	Jul.	1	30s				
1997	Jul.	1	1999	Jan.	1	31s				
1999	Jan.	1	2006	Jan.	1	32s				
2006	Jan.	1	2009	Jan.	1	33s				
2009	Jan.	1				34s				

SOURCES OF UT1

As pointed out earlier, the Earth's variable rate of rotation makes it necessary to determine UT1–UTC observationally using astronomical and geodetic techniques. However, for low-accuracy applications some might choose to ignore the difference between UT1 and UTC, and effectively set UT1–UTC = 0 for all time, assuming that the current definition of UTC will ensure that |UT1–UTC| < 0.9s (13.5 seconds of arc).

However, many users can benefit from the use of the more accurate data that have been provided by international service agencies. Figure 2 shows these observational values as a function of time. It demonstrates the effect of the definition of UTC in place since 1970 whereby UTC is adjusted through the use of leap seconds.

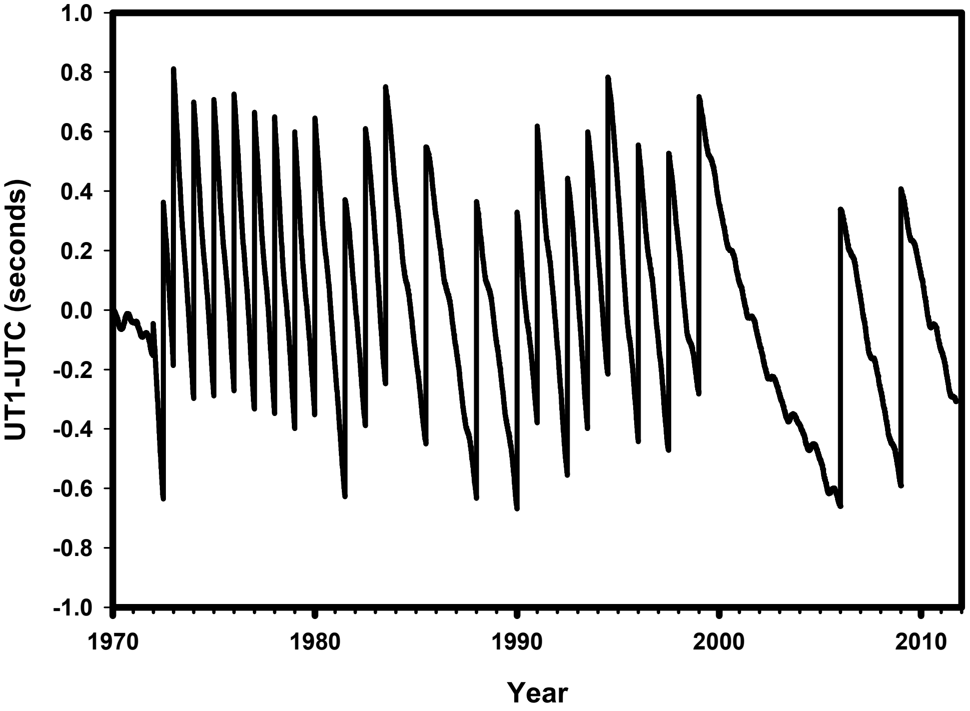

Figure 2. Observed values of UT1–UTC.

The IERS provides a variety of sources for UT1 that can be obtained quite readily. It was established as the International Earth Rotation Service in 1987 by the International Astronomical Union and International Union of Geodesy and Geophysics. In 2003 it was renamed to the International Earth Rotation and Reference Systems Service. The IERS serves astronomical, geodetic and geophysical communities by providing:

- International Celestial Reference System (ICRS) and its realization, the International Celestial Reference Frame (ICRF),
- International Terrestrial Reference System (ITRS) and its realization, the International Terrestrial Reference Frame (ITRF),
- Earth orientation parameters required to transform between the ICRF and the ITRF and for research,

- Geophysical data to interpret time/space variations in the ICRF, ITRF or earth orientation parameters, and model such variations, and
- Standards, constants and models (*i.e.*, conventions) encouraging international adherence.

In partial fulfillment of its mission, then, the IERS provides UT1–UTC in a variety of forms. All are supplied in electronic and paper formats. Definitive observed values with errors less than 10 μs (0.000 15 seconds of arc) are made available about one month after the observations were made. Rapid Service values with accuracy between 20 and 30 μs are also provided with a latency of a few days. In addition, predicted values are supplied for short terms (< 1 year in advance) and for long term (up to twenty years in advance). Obviously the errors of those predictions increase with the length of the forecast, and Figure 3 shows the maximum error that can be expected in IERS products in comparison with the maximum error achieved by assuming that UT1–UTC can be ignored. These predictions are based on the most recent astronomical and meteorological data as well as statistical models.

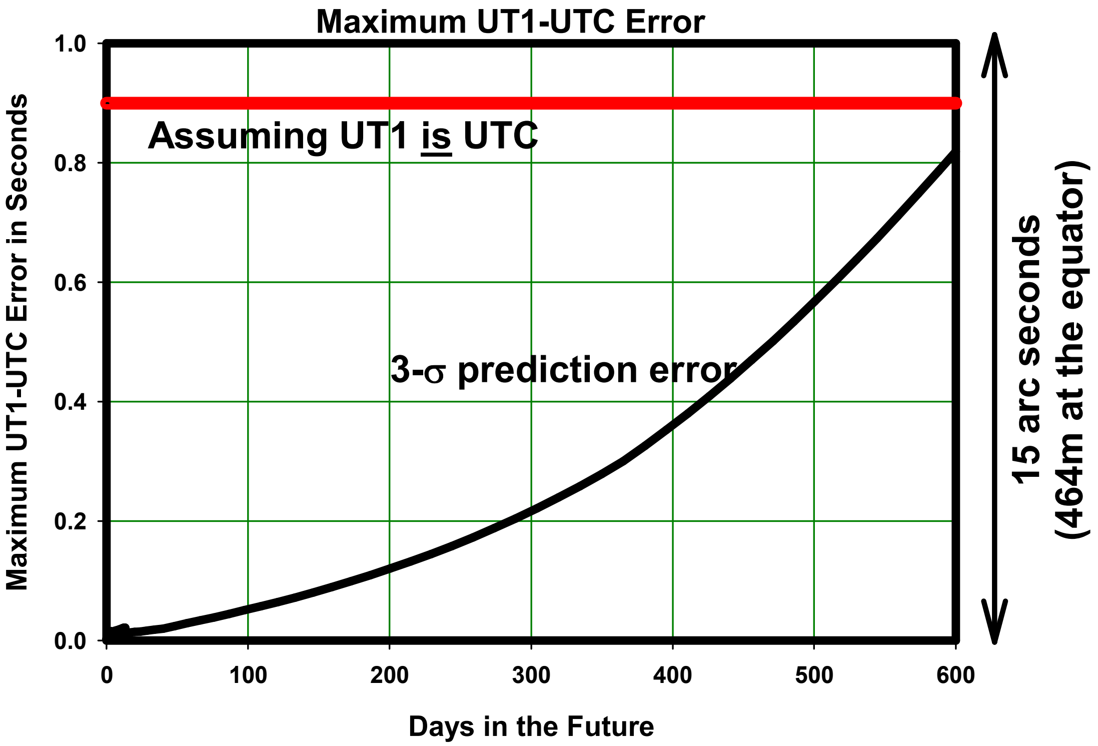

Figure 3. Maximum error in IERS estimates of UT1–UTC.

The effects of daily and sub-daily variations in the Earth's rotation are not included in the UT1–UTC values provided by the IERS. These very small additional components should be added to the UT1–UTC values derived after interpolating the daily estimates for the epoch of interest to account for the effects of ocean tides and libration. The procedures to do this are provided in the IERS Conventions 2010.

In addition to the products provided directly by the IERS Product Centers it is expected that UT1–UTC values will be part of the navigational message broadcast by the GPS III satellites. That would make accurate values of UT1–UTC easily accessible to users with the appropriate receivers. If there were sufficient user demand, we might even envision a UT1–UTC application being made available for future hand-held devices.

DISCUSSION

UTC was defined in 1970 so as to ensure that |UT1–UTC| < 0.9s, and consequently some applications have made use of this definition to neglect the effect of non-zero values of UT1–UTC. When the definition was originally formulated in 1970, it was designed principally to accommodate the needs of celestial navigators.[7] Since that time celestial navigation has been largely surpassed by electronic navigation and the requirement for celestial navigation is no longer so critical.

Knowledge of the Earth Rotation Angle, however, remains an important requirement for many uses in geodesy, astronomy and particularly in various space applications. In these fields the need for improved accuracy continues to grow, and it is likely that it will continue to grow in the future. The fifteen arcsecond accuracy provided by assuming that UTC can be used as a proxy for UT1 is unlikely to meet those future requirements. To meet those needs it is important to plan for ways not only to improve observations, but also for ways to provide this information to users.

UT1–UTC estimates are now available routinely at the level of accuracy of ±0.000 15 seconds of arc as opposed to the ±0.015 seconds of arc available in 1970. At that time UT1–UTC estimates were available via paper bulletins one month after the astronomical observations. Now they are available electronically in near real-time. These facts would suggest that those applications that require knowledge of the relationship between celestial and terrestrial reference systems be re-investigated to determine if they can benefit from the accuracy and availability of UT1–UTC estimates provided routinely by the IERS. Significant improvements may be possible if actual observations were used as opposed to using UTC as a proxy for UT1.

CONCLUSION

Relating celestial and terrestrial reference systems requires knowledge of the Earth’s rotation angle in space. This information is proved by a mathematical expression for the Earth Rotation Angle that requires numerical values of the angle UT1. Those data can be obtained by combining estimates of UT1–UTC with Coordinated Universal Time (UTC). The difficulty in predicting the Earth’s variable rate of rotation makes it necessary for UT1–UTC to be determined observationally and for those values, as well as forecast values, to be disseminated in a convenient way for access by users.

Currently low-accuracy (±15″) values of UT1 can be obtained by ignoring the difference between UT1 and UTC. However, high-accuracy (±0.000 15″) values are also routinely available electronically in near real time at no cost to the user. It is likely that the demand for high-accuracy will grow in the future, and international and national agencies are prepared to meet that need. Applications requiring UT1 data might benefit from investigating potential improvements to operations made possible by taking advantage of high-accuracy estimates of UT1–UTC instead of just using UTC.

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